

Submesoscale Frontogenesis Drives Strong Finescale Tracer Subduction

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Key Points:

- Large-eddy simulations reveal enhanced finescale tracer subduction at submesoscale fronts relative to non-frontal convective regions
- Finescale motions near sharp fronts drive about 20% of the total subduction, despite occupying only about 7% of the frontal zone area
- Strong finescale subduction at fronts is primarily driven by frontogenesis via horizontal shear production, not by surface forcing

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Abstract Vertical exchange between the surface mixed layer (ML) and the interior is important for biogeochemistry and air–sea interaction. Focused observations have consistently shown that submesoscale processes (horizontal scales of 0.1–10 km) generate substantial exchange of heat, freshwater, and biogeochemical tracers across the ML base, particularly near fronts. However, the role of finescale turbulence in this exchange remains poorly understood. Here we investigate this multiscale problem by analyzing the transport of a surface-fluxed passive tracer in large-eddy simulations of baroclinic mixed-layer instabilities (MLIs) under surface cooling. Budgets for the tracer below the evolving ML show that finescale turbulence contributes to tracer subduction only at fronts sharpened to a few hundred meters wide by eddy straining and horizontal convergence. Although submesoscale vertical flux dominates the exchange across the ML base, finescale turbulent flux is comparable to the indirect contribution from MLI-induced restratification. Analysis of the finescale energetics shows that the turbulence at sharp fronts is primarily driven by shear production associated with frontogenesis, with large contributions from horizontal shear, rather than by convective forcing. In contrast, outside the frontal zone turbulence is driven solely by surface forcing, and the tracer is largely confined to the ML. This suggests a multiscale picture of vertical exchange at submesoscale fronts that is not represented in existing parameterizations. Submesoscale-permitting ocean models may partially capture the frontogenetic process, but the associated forward energy flux is likely dissipated implicitly or explicitly, rather than contributing to vertical mixing and tracer transport.

Plain Language Summary Exchange of tracers such as heat, nutrients, and carbon between the surface ocean and the deeper interior affects climate and marine ecosystems. Observations and models have shown that small ocean fronts (0.1–10 km wide) can greatly enhance this vertical exchange. However, the role of even smaller (less than 100 m), but faster, turbulence is not well understood. Here we use turbulence-resolving simulations of ocean fronts to analyze the evolution of a tracer injected at the surface (like a dye). We find that turbulent motions are strongly intensified at sharp fronts, providing an additional, significant contribution to tracer transport into the interior. This turbulence is mainly driven by strong spatial differences of currents as the front sharpens, rather than by surface forcing. These results reveal a complex, multiscale process that is not fully captured in coarse-resolution ocean models, underscoring the need for model improvements.

1. Introduction

Surface–interior exchange provides a two-way pathway between the mixed layer (ML) and the deeper interior: subduction moves surface-rich tracers (e.g., anthropogenic heat, oxygen, suspended particulate organic carbon) into the interior, where they can be sequestered from the atmosphere for extended periods; meanwhile, entrainment draws interior-rich tracers (e.g., nutrients, dissolved inorganic carbon) into the ML, where they can stimulate phytoplankton blooms and alter surface pCO₂. Here, subduction and entrainment are defined as the transport of tracer out of and into the ML, respectively, regardless of the underlying physical mechanisms. Accurately characterizing the vertical exchange is therefore essential for predictions of oxygen ventilation, carbon export, primary production, air–sea interaction, and a range of other processes. The rate of this exchange is fundamentally controlled by vertical fluid motions, which have a wide range of spatial and temporal scales. Early work focused on the vertical pumping by mesoscale eddies to close large-scale tracer budgets (McGillcuddy, 2016), but it later became clear that smaller submesoscale flows (spatial scales of $\mathcal{O}(0.1\text{--}10\text{ km})$ and timescales of days) also play a significant role, because they have stronger vertical velocities due to weaker

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rotational and stratification constraints (Lévy et al., 2018; Mahadevan, 2016). Vertical velocities are further intensified at even finer scales (spatial scales of $\mathcal{O}(1\text{--}100\text{ m})$ and timescales of minutes to hours), where the flow transitions to nonhydrostatic, nearly isotropic, three-dimensional (3D) turbulence. Both the submesoscale and finescale processes are effectively subgrid for most global ocean and climate models, and thus must be parameterized to capture their impacts on large-scale tracer evolution over long timescales. These two processes also coexist in the ML, and can interact in complicated ways to shape their net effects on upper-ocean energy transfer (Taylor & Thompson, 2023) and vertical exchange. The joint impacts of submesoscale processes and finescale turbulence on vertical exchange are not fully understood (Mahadevan et al., 2020). Addressing this gap is the motivation for the present study.

Baroclinic mixed-layer instability (MLI, Boccaletti et al., 2007) is considered as a major mechanism for submesoscale vertical transport, though other processes—such as symmetric instability (Brannigan, 2016)—may also contribute. MLIs develop along ML density fronts, generating cross-frontal overturning circulations that slump the fronts and advect tracers primarily along isopycnals (Fox-Kemper et al., 2008). The frontal slumping can indirectly subduct tracers by shoaling the ML (i.e., restratification). The resulting ML eddies may grow in vertical scale (K. S. Smith & Vallis, 2001) during frontal slumping, boosting the trapping of fluid in stratified layers below (Lo Piccolo, 2026). As MLIs evolve into ML eddies, they also vigorously stir tracers along isopycnals and thus enhance isopycnal diffusion (Badin et al., 2011; Cao et al., 2026; Jaeger et al., 2020). This direct advection and diffusion along sloping isopycnals project onto the vertical, generating vertical transport that extends below the ML base. However, this vertical flux is not captured by existing MLI parameterizations (Bodner et al., 2023; Fox-Kemper & Ferrari, 2008), which were designed to represent restratification only. While efforts to address this deficiency are underway (Lo Piccolo, 2026), important questions remain regarding the role of finescale turbulence in surface–interior exchange, given the multiscale nature of the flow at ML fronts (Hamlington et al., 2014; Johnson & Fox-Kemper, 2024; Sullivan & McWilliams, 2024).

A substantial body of evidence for strong submesoscale vertical transport comes from studies employing large-scale Reynolds-averaged Navier–Stokes (RANS) simulations, where the effect of finescale turbulence is parameterized. These simulations have shown enhanced tracer subduction (Balwada et al., 2018) and stronger upward nutrient supply (Lévy et al., 2012; Uchida et al., 2020) as horizontal grid resolution increases to permit more submesoscale activity (Brett et al., 2023; Richards et al., 2021). Lagrangian analysis of particles released in submesoscale-permitting simulations (Freilich & Mahadevan, 2021) also revealed that the subduction pathways are coherent and concentrated along strong submesoscale fronts. On the observational side, many studies have documented signatures of submesoscale vertical transport near individual fronts, typically manifesting as nearly along-isopycnal tracer filaments that extend across the ML base (Archer et al., 2020; Hosegood et al., 2013; Omand et al., 2015; Ruiz et al., 2019). Although these features are qualitatively consistent with RANS simulation results, it remains unclear whether the associated vertical transport can be attributed solely to submesoscale processes, particularly given evidence that existing one-dimensional (1D) mixing parameterizations misrepresent finescale mixing at submesoscale fronts (Bachman et al., 2017; Johnson & Fox-Kemper, 2024; Zheng et al., 2025).

The importance of finescale turbulence for vertical transport has long been recognized, but the current understanding is largely limited to horizontally homogeneous settings. There, fluxes across the ML base are dominated by intermittent plume-like structures (Johnston & Rudnick, 2009; Polton & Belcher, 2007), fueled by a combination of downward transport of kinetic energy (KE) produced near the surface and local shear production near the base (Q. Li & Fox-Kemper, 2020). At fronts, submesoscale restratification tends to suppress surface-forced turbulence by limiting its vertical extent, at the same time, submesoscale processes can also generate turbulence through cross-scale KE transfer (Molemaker et al., 2010; Skillingstad & Samelson, 2012; Taylor & Ferrari, 2010; Verma et al., 2019; Zheng et al., 2025). Both effects are believed to be of global significance (Dong et al., 2024; Johnson et al., 2016), and hence could influence the role of finescale turbulence in surface–interior exchange in a manner not currently well understood.

Since measuring multiscale vertical transport in the field is notoriously difficult, turbulence-resolving large-eddy simulation (LES) has emerged as a powerful tool (e.g., Bo et al., 2026; Hamlington et al., 2014; Skillingstad et al., 2017; Sullivan & McWilliams, 2024; Taylor, 2016; Taylor & Ferrari, 2010; Verma et al., 2019, 2022). K. M. Smith et al. (2016) investigated vertical tracer transport in a $20 \times 20\text{ km}$ LES of submesoscale fronts forced by surface winds and waves (see also Hamlington et al., 2014). The simulated flow lies in a regime where the

strength of surface-forced turbulence is comparable to MLI-induced restratification. By employing a set of tracers with various initial and boundary conditions, their analysis revealed a complex dependence of total tracer transport on the tracer initialization, highlighting the need for further study. Nevertheless, a common result across all tracers is that the spatially averaged finescale vertical tracer flux is generally suppressed within regions where submesoscale eddies are active. Although they argued this suppression is primarily due to submesoscale restratification, it is unclear if this suppression could be fully explained by the ML shoaling—particularly as Johnson and Fox-Kemper (2024) showed that the suppression of vertical mixing exceeds the expectations based on Monin and Obukhov (1954) scaling.

More recently, Pham et al. (2024) used Lagrangian particles in a 6×18 km LES of submesoscale fronts forced by surface cooling to specifically investigate the subduction pathways. In a similar flow regime to that of K. M. Smith et al. (2016), they found that the submesoscale flow and finescale turbulence act in concert to achieve rapid coherent subduction. Statistics of particle velocities suggest that subducted particles are first advected laterally by submesoscale currents toward preferred locations, where they undergo intense downwelling across the ML base before being carried deeper into the pycnocline by much slower submesoscale vertical motions. This intriguing result implies that selectively intensified turbulence in the frontal region may be critical in setting the rate of subduction, but its precise contribution cannot be readily quantified using particle-based analysis. Furthermore, the importance of finescale turbulence for surface–interior exchange in other flow regimes remains underexplored.

Here, we focus on a flow regime that is strongly influenced by submesoscale restratification. This commonly occurs during spring and is likely important for carbon export because restratification often triggers phytoplankton bloom (Mahadevan et al., 2012; Omand et al., 2015). Although a few previous studies (Callies & Ferrari, 2018; Taylor, 2016; Verma et al., 2022) have investigated the weakly forced regime, this work complements those by examining the vertical transport of a surface-fluxed tracer in freely evolving fronts. Specifically, we show that the mean finescale tracer flux at fronts is generally weaker than that outside the fronts (consistent with K. M. Smith et al., 2016); however, it penetrates deeper, indicating net finescale transport out of the ML at fronts. The identified finescale subduction echoes the results of Pham et al. (2024), and the contribution of finescale turbulence to interior tracer accumulation is about 20% of the total, despite representing only approximately 7% of the frontal zone area. The enhanced turbulence and finescale tracer flux near sharp fronts are primarily driven by shear production associated with frontogenesis, with the largest contribution from horizontal shear, rather than by surface forcing. These results suggest a new mechanism for surface–interior exchange at submesoscale fronts, and offer a potential reinterpretation of the turbulent subduction shown in Pham et al. (2024).

This manuscript is structured as follows. Sections 2 and 3 give the preliminaries: a description of the numerical simulations, their spectral properties, and the scale decomposition. The main results are presented in Section 4, including an overview of the flow field, mean flux profiles, spatial characteristics of vertical tracer fluxes, volume budgets for the interior tracer, and KE budgets for turbulence near sharp fronts. The implications for parameterization are discussed in Section 5, followed by the conclusions in Section 6.

2. Numerical Simulations

The transport of a passive tracer during the spindown of two submesoscale density fronts under surface buoyancy loss is studied using LES of the nonhydrostatic incompressible Boussinesq equations:

$$\frac{Du}{Dt} = -\nabla p - f\hat{k} \times u + b\hat{k} - \nabla \cdot S^u, \quad (1)$$

$$\nabla \cdot u = 0, \quad (2)$$

$$\frac{Db}{Dt} = -\nabla \cdot S^b, \quad (3)$$

$$\frac{Dc}{Dt} = -\nabla \cdot S^c, \quad (4)$$

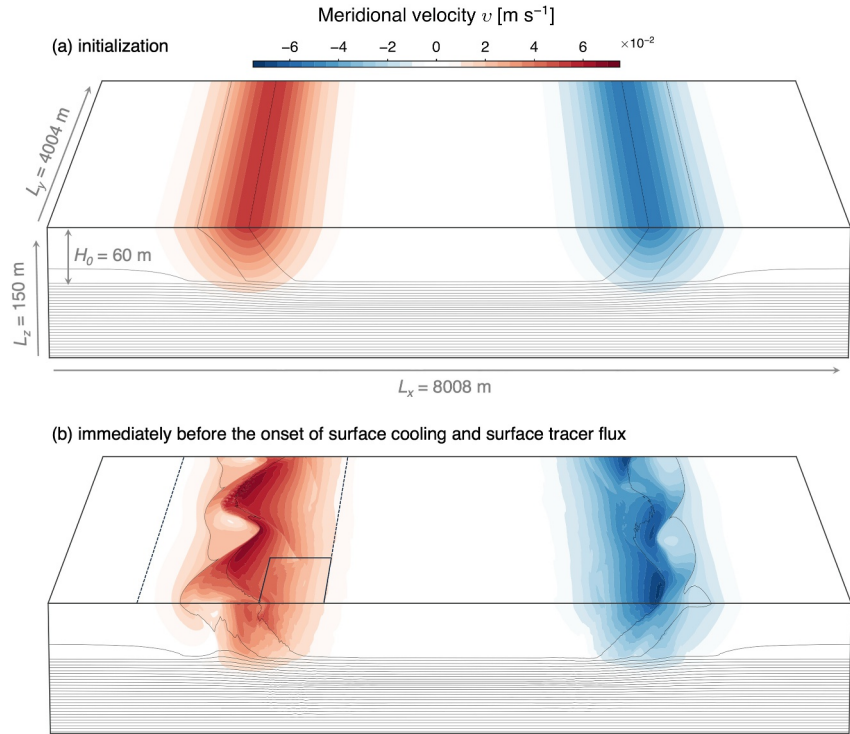


Figure 1. Overview of the frontal jets at (a) the time of initialization and (b) immediately before the onset of surface cooling and surface tracer flux. The meridional velocity v (colored shading) is in thermal wind balance with the buoyancy b (contours with interval $\Delta b \approx 2.35 \times 10^{-5} \text{ m s}^{-2}$) at initialization. Areas outlined by dashed lines and solid lines in panel (b) show the location of the subdomain and cutout in Figure 3, respectively.

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ is the material derivative, $\mathbf{u} = (u, v, w)$ is the 3D velocity vector, p is the kinematic pressure, f is a constant Coriolis frequency (the unit vector $\hat{\mathbf{k}}$ denotes the upward vertical direction), b is the buoyancy, and c is the tracer concentration (unitless, varying from 0 to 1). The subgrid-scale (SGS) fluxes of momentum, buoyancy, and tracer ($\mathcal{S}^u$, $\mathcal{S}^b$, and $\mathcal{S}^c$) are modeled using a constant-coefficient Smagorinsky–Lilly closure, in which the SGS Prandtl and Schmidt numbers are set to unity. These governing equations are integrated forward using the Julia package Oceananigans (Ramadhan et al., 2020; Wagner et al., 2025). We use a fifth-order weighted essentially non-oscillatory advection scheme and a third-order Runge–Kutta time-stepping method, where the time step is adapted according to a fixed Courant number of 0.9. For ease of discussion, buoyancy will be occasionally referred to as “temperature.”

The computational domain (Figure 1) is a horizontally periodic box with size $(L_x, L_y, L_z) = (8008, 4004, 150) \text{ m}$ in the zonal, meridional, and vertical direction. The grid spacing is uniform in the horizontal ($\Delta x = \Delta y = 3.64 \text{ m}$), but is stretched in the vertical from about 0.92 m near the surface to about 4.66 m near the bottom. This grid spacing is comparable to previous studies (Hamlington et al., 2014; Taylor, 2016; Verma et al., 2022). A resolution test with the grid spacing enlarged by 51% (in all three dimensions) yields differences of less than 10% in fields such as tracer concentration, finescale vertical buoyancy and tracer fluxes. No normal flow and no-flux boundary conditions are used at the surface except for the tracer and buoyancy, which have prescribed fluxes (see below). A 30 m thick sponge layer at the bottom relaxes velocity and buoyancy to their initial state to prevent wave reflection. The initial two-front configuration (Figure 1a) is constructed by superimposing a wide warm filament onto a two-layer stratification profile,

$$b = \frac{M_0^2 L_f}{2} \left\{ \tanh \left[\frac{2(x + L_x/4)}{L_f} \right] - \tanh \left[\frac{2(x - L_x/4)}{L_f} \right] \right\} \Lambda(z) + b_n(z), \quad (5)$$

$$b_n = N_1^2(z + L_z) + (N_0^2 - N_1^2) \max(z + H_0, 0), \quad (6)$$

Table 1
Parameters of Numerical Simulations Used in This Study

Parameter	Symbol	Value
Coriolis frequency	f	$1 \times 10^{-4} \text{ s}^{-1}$
Initial upper layer thickness	H_0	60 m
Initial stratification (upper, lower)	(N_0^2, N_1^2)	$(3.6, 72) \times 10^{-7} \text{ s}^{-2}$
Initial maximum horizontal buoyancy gradient	M_0^2	$6 \times 10^{-8} \text{ s}^{-2}$
Initial frontal width	L_f	1,000 m
Initial Rossby number	$Ro = \frac{M_0^2 H_0}{f^2 L_f}$	0.36
Initial balanced Richardson number	$Ri_B = \frac{N_0^2 f^2}{M_0^2}$	1
Surface buoyancy flux	B_0	$[0, 4.79, 47.9] \times 10^{-10} \text{ m}^2 \text{ s}^{-3}$
Restratification ratio	R_{MLI}	$[0, 0.004, 0.04]$
Potential energy anomaly threshold	ΔE_p	$[-, 0.15, 4.74] \text{ J m}^{-2}$

Note. Values in [] correspond to different simulations.

$$\Lambda = \frac{1}{2} \left\{ 1 + \tanh \left[\frac{6(z + 3H_0/2)}{H_0} \right] \right\}, \quad (7)$$

where b_n gives a piecewise profile with constant stratification N_0^2 and N_1^2 for the upper and lower layer, respectively (generally $N^2 = \partial b / \partial z$); H_0 is the upper layer thickness, M_0^2 is the magnitude of initial maximum horizontal buoyancy gradient (generally $M^2 = |\nabla_h b|$ with ∇_h the horizontal gradient operator), and L_f is the initial frontal width. The nondimensional function Λ describes the vertical structure of the filament, which is uniform in the upper layer and extends to about $\frac{3}{2}H_0$ below the surface. The initial velocity is zero everywhere except at the fronts, where two meridional jets (Figure 1a) are established by vertically integrating the thermal wind balance from zero at the bottom.

Values of simulation parameters are summarized in Table 1. With these frontal parameter values, linear stability theory (Boccaletti et al., 2007; Stone, 1970) suggests that the fastest growing mode of MLI has a length scale of approximately 2 km, and a growth time scale of about 0.5 days. Following the introduction of small-amplitude perturbations near the surface, we conduct an unforced base simulation without tracer for approximately 7 days to allow the MLI to reach finite amplitude (Figure 1b). At early times of the unforced run, centrifugal-symmetric instabilities (Chor et al., 2022; Thomas et al., 2013) and other mixed modes (Stamper & Taylor, 2016) develop on the warm side of the front because of the narrow frontal width and small balanced Richardson number in the upper layer (initial $Ri_B = 1$). They decay quickly after the potential vorticity is brought back to zero around day 5 and do not impact the results presented below.

A surface-fluxed passive tracer is used to study the multiscale surface–interior exchange. This idealized tracer mimics biogeochemical tracers such as carbon and oxygen, and is known to yield results that are less sensitive to setup details than tracers released at a fixed depth (K. M. Smith et al., 2016). On day 7 of the base run, we impose an upward (i.e., ocean cooling) surface buoyancy flux B_0 , and a downward surface tracer flux C_0 (defined below) with initial tracer concentration set to zero everywhere, and continue the simulation for another 60 hr. For convenience, we will refer to the time of tracer release as hour 0 hereafter. This initialization strategy is adopted to avoid the complexities introduced by other frontal instabilities and to better control the flow regime. Applying the forcing earlier could alter submesoscale restratification, eddy KE, and the evolution of frontal intensity (Dauhajre et al., 2025). These effects warrant further investigation in future studies.

Three cases are considered, each with a different surface buoyancy flux (see Table 1), corresponding to no cooling, weak cooling, and moderate cooling. The equivalent surface heat flux magnitude is 0, 1, and 10 W m^{-2} . These surface buoyancy flux magnitudes are somewhat small compared to typical observational values, but it is the relative strength of surface forcing and restratification that governs the flow regime. Generally speaking, the competition between the restratifying effect of MLI and the destratifying effect of turbulent mixing due to surface

forcing can be assessed by the restratification ratio (Mahadevan et al., 2010). Here we only have surface buoyancy forcing, so

$$R_{\text{MLI}} = \frac{B_0}{M_0^4 H_0^2 f}. \quad (8)$$

Note that we have omitted the scaling coefficient ($c_e \approx 0.06\text{--}0.08$) for the MLI-induced vertical buoyancy flux (Fox-Kemper et al., 2008) in the denominator. Thus, when $R_{\text{MLI}} \ll c_e$, MLI-induced restratification is expected to dominate; whereas for $R_{\text{MLI}} \gg c_e$, turbulent mixing is expected to overcome restratification. With $R_{\text{MLI}} = 0.004$ and 0.04 , the two forced simulations in our study sample two contrasting flow regimes: the weak-cooling case represents idealized spring conditions when the surface-forced turbulence diminishes but the submesoscale restratification remains strong; the moderate-cooling case represents idealized early-fall conditions when the surface-forced turbulence becomes comparable to the submesoscale restratification. For reference, the surface forcing in the simulations analyzed by K. M. Smith et al. (2016) and Pham et al. (2024) were relatively stronger ($R_{\text{MLI}} \approx 0.5, 0.1$, respectively).

For each simulation, the surface buoyancy flux is kept as a constant, but the surface tracer flux depends on the surface concentration c_{surf} , namely,

$$C_0 = -k_c (c_{\text{air}} - c_{\text{surf}}), \quad (9)$$

where $c_{\text{air}} = 1$ represents an effectively infinite reservoir (or rapid refreshing) of tracer in the air above the ocean surface, k_c is the tracer flux rate, or piston velocity, in the case of air–sea gas transfer. The piston velocity is commonly parameterized as a function of surface forcing, and typically ranges from $\mathcal{O}(10^{-5})$ to $\mathcal{O}(10^{-4})$ m s^{−1} for gases such as oxygen and carbon dioxide (Wanninkhof, 2014). K. M. Smith et al. (2016) examined a suite of surface-fluxed tracers with k_c spanning $\mathcal{O}(10^{-6})$ to $\mathcal{O}(10^{-2})$ m s^{−1}. Although the spatial pattern and vertical flux magnitude of these tracers differ substantially, the underlying transport efficiency and the relative contribution by submesoscale flow and finescale turbulence show little variation across tracers. Based on this, we adopt a flux rate ($k_c = 1 \times 10^{-3}$ m s^{−1}) that is higher than realistic values, because our focus is on how tracer is transported downward from the ML. This increased injection rate helps quickly flux tracer into the near-surface waters, thereby supplying the source to track deeper transport in short-duration simulations. The chosen piston velocity is comparable in magnitude to the vertical velocity scale of convection (Deardorff, 1970), but is an order of magnitude larger than that of MLI-induced overturning circulation (Fox-Kemper et al., 2008).

The ML depth is diagnosed from potential energy (PE) anomaly (Reichl et al., 2022). This approach measures bulk stratification and provides an energetically consistent definition of ML that is more robust than conventional buoyancy-threshold criteria (Treguier et al., 2023). This is particularly important at fronts, where the near-surface layer often remains stratified. Physically, the PE anomaly represents the energy required to homogenize the water column to a certain depth, and therefore is generally a monotonic function with depth. The ML depth H is identified as the depth where the PE anomaly first exceeds a threshold value ΔE_p , such that

$$\Delta E_p = \int_{-H}^0 (\rho_m - \rho) g z dz, \quad (10)$$

where ρ_m is the mean potential density over the depth range $[-H, 0]$, and $\rho = \rho_0(1 - b/g)$ is the potential density, with $\rho_0 = 1,026$ kg m^{−3} as the reference density. The filtered buoyancy (Section 3) is used for this calculation to avoid noise from turbulent fluctuations. We pick a threshold value that is slightly higher than the amount of energy available for mixing from surface forcing (Reichl & Hallberg, 2018), so the exact value of ΔE_p increases with surface cooling (Table 1), and the resulting ML depth is slightly deeper than the boundary layer depth where turbulent fluxes vanish. This choice helps to highlight the deep-reaching finescale tracer flux at fronts.

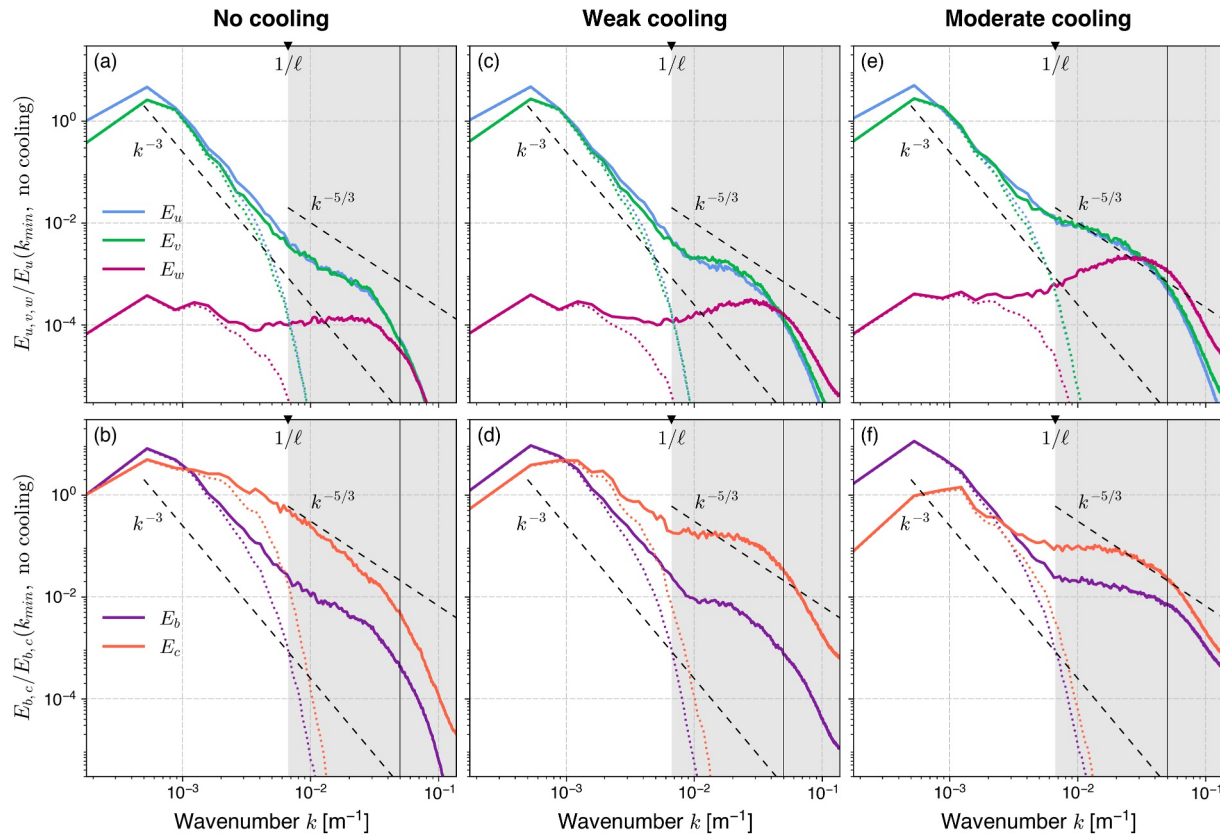


Figure 2. Circularly integrated horizontal wavenumber spectra of (a, c, and e) velocity component u , v , w , (b, d, and f) buoyancy b and tracer concentration c at 5 m depth, hour 31 of each simulation. Gray shading area indicates scales smaller than the filter scale ($\ell = 150$ m). Dotted lines are the corresponding spectra of filtered fields. Vertical lines show the effective resolution ($\approx 5\Delta x$).

3. Spectral Characteristics and Scale Decomposition

To separate the transport by finescale turbulence from that by submesoscale motions, one can either use Reynolds averaging (Johnson & Fox-Kemper, 2024) or filtering (Hamlington et al., 2014; K. M. Smith et al., 2016; Verma et al., 2022). Here we adopt spatial filtering, as this approach preserves local information and provides a mathematically coherent way to study multiscale problems within the coarse-graining framework (Aluie et al., 2018). At each depth and time, the total field of an arbitrary variable ϕ is decomposed into a submesoscale component $\bar{\phi}$, and a finescale component ϕ' , where the overbar denotes filtering and the prime denotes residual. We use a two-dimensional (2D) symmetric Gaussian filter implemented in the python package GCM-Filters (Grooms et al., 2021; Loose et al., 2022), which while not spectrally sharp, is preferred because it avoids ringing in physical space (which could obscure spatial structures) and prevents spurious negative values in positive semi-definite fields (e.g., tracer, variance, KE).

The GCM-Filters package defines the Gaussian filter scale ℓ as $\sqrt{12}$ times the standard deviation of the Gaussian kernel to achieve the same effect on filtered field as a boxcar filter with width ℓ . The choice of the filter scale ($\ell = 150$ m) for flow decomposition is determined from the horizontal wavenumber spectra, which are computed separately for two square half-domains and then averaged. Figure 2 shows the near-surface (5 m depth) velocity, buoyancy, and tracer spectra from a representative snapshot (hour 31) of each simulation. Temporal variability of these spectra is only significant during the first several hours of the simulation as the spectra adjust to the onset of convective forcing and surface tracer flux. In all three simulations, the low-wavenumber part of the horizontal velocity and buoyancy spectra roughly scales as k^{-3} , consistent with quasigeostrophic theory (Charney, 1971). This is a steeper slope over the submesoscales than other simulations (Hamlington et al., 2014), which scale as k^{-2} , perhaps because the largest eddies there are considerably smaller than the domain size. Potentially as a result of nonhydrostatic effects, the vertical velocity spectra are nearly flat in the low-wavenumber range, contrasting to

the continuity-implied k^{-1} shape often seen in submesoscale-permitting, hydrostatic RANS simulations (Freilich & Mahadevan, 2021). Nevertheless, the large-scale KE is still dominated by horizontal motions, indicating anisotropic, quasi 2D submesoscale currents (Hamlington et al., 2014). Similar to the results of K. M. Smith et al. (2016), the slope of tracer spectra at low wavenumbers is shallower than that of horizontal velocity and buoyancy. This is likely because the large-scale eddy energy is mostly injected at scales of MLI [$\mathcal{O}(0.1\text{--}1\text{ km})$], whereas the tracer variance is also injected at smaller scales. In fact, for initial finite source tracers without surface fluxes, the spectral slope is close to that of horizontal velocity and buoyancy (e.g., see Figure 10 of K. M. Smith et al., 2016). At higher wavenumbers, these spectra gradually transition to nearly isotropic, quasi 3D cascade with a $k^{-5/3}$ slope (Corrsin, 1951; Kolmogorov, 1941) before grid-scale effects attenuate variance around the effective resolution ($\approx 5\Delta x$).

As the surface buoyancy flux increases, more vertical KE is injected at small scales through the buoyancy production of gravitational instability, and the velocity spectra become more isotropic, especially at deeper depths. The increase of surface cooling also reduces tracer variance at large scales; this may be related to enhanced downscale transfer of tracer variance in the moderate-cooling case. For all three simulations, there exists a separation scale that approximately splits the spectra into two different regimes corresponding to quasi 2D and quasi 3D dynamics, respectively. Although the separation scale appears to increase slightly with stronger surface cooling, we adopt a common filter scale ($\ell = 150\text{ m}$) for consistency and simplicity. Therefore, results from the filtering in different simulations can be directly compared in the context of a RANS model with grid resolution of about ℓ . Filter scales of 100 and 200 m were also tested and found to have no significant impact on our main conclusions, apart from minor variations in the precise partitioning between submesoscale and finescale fluxes.

The Gaussian kernel is spatially invariant (i.e., homogeneous), so the filtering operation commutes with derivatives. Applying the filter to Equations 1–4 yields the filtered equations in flux form (given that $\nabla \cdot \bar{\mathbf{u}} = 0$):

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} = -\nabla \cdot \bar{\boldsymbol{\tau}} - \nabla \bar{p} - f \hat{\mathbf{k}} \times \bar{\mathbf{u}} + \bar{b} \hat{\mathbf{k}} - \nabla \cdot \boldsymbol{\tau}', \quad (11)$$

$$\frac{\partial \bar{b}}{\partial t} = -\nabla \cdot \bar{\mathbf{B}} - \nabla \cdot \mathbf{B}', \quad (12)$$

$$\frac{\partial \bar{c}}{\partial t} = -\nabla \cdot \bar{\mathbf{F}} - \nabla \cdot \mathbf{F}', \quad (13)$$

where the SGS terms are neglected as they are generally small away from the surface. The first term on the right-hand side of each equation above is the submesoscale flux divergence, representing the advection by the submesoscale flow $\bar{\mathbf{u}}$, that is, $\bar{\boldsymbol{\tau}} = \bar{\mathbf{u}}\bar{\mathbf{u}}$, $\bar{\mathbf{B}} = (\bar{\mathbf{B}}_h, \bar{\mathbf{B}}_v) = \bar{\mathbf{u}}\bar{b}$, $\bar{\mathbf{F}} = (\bar{\mathbf{F}}_h, \bar{\mathbf{F}}_v) = \bar{\mathbf{u}}\bar{c}$, where the subscript h and v denote horizontal and vertical component, respectively. Equations 11–13 have the same structure as the unfiltered equations, except for an additional term representing the divergence of finescale flux (or stress):

$$\mathbf{F}' = (\mathbf{F}'_h, \mathbf{F}'_v) = \bar{\mathbf{u}}\bar{c} - \bar{\mathbf{u}}\bar{c}, \quad (14)$$

$$\mathbf{B}' = (\mathbf{B}'_h, \mathbf{B}'_v) = \bar{\mathbf{u}}\bar{b} - \bar{\mathbf{u}}\bar{b}, \quad (15)$$

$$\boldsymbol{\tau}' = \begin{bmatrix} \tau_{uu} & \tau_{uv} & \tau_{uw} \\ \tau_{uv} & \tau_{vv} & \tau_{vw} \\ \tau_{uw} & \tau_{vw} & \tau_{ww} \end{bmatrix}, \quad (16)$$

where $\tau_{uv} = \overline{uv} - \bar{u}\bar{v}$, and similarly for other entries of the symmetric tensor $\boldsymbol{\tau}'$.

The submesoscale flux can be further decomposed; using the vertical tracer flux as an example:

$$\bar{\mathbf{F}}_v = \bar{w}\bar{c} = \langle \bar{w} \rangle \langle \bar{c} \rangle + \langle \bar{w} \rangle \bar{c}^s + \bar{w}^s \langle \bar{c} \rangle + \bar{w}^s \bar{c}^s, \quad (17)$$

where $\langle \rangle$ denotes horizontal averaging over an area with active submesoscale eddies and fronts (the frontal zone, defined in Section 4.1), and the superscript s denotes eddy perturbation relative to the horizontal mean, for example, $\bar{w}^s = \bar{w} - \langle \bar{w} \rangle$. The first term in Equation 17 is negligible because $\langle \bar{w} \rangle$ is nearly zero (this argument does not apply to submesoscale horizontal fluxes). The second and the third term vanish by definition upon horizontal averaging, although the third term can be locally significant. The last term dominates the spatial mean and is thus termed the submesoscale eddy flux, as it represents the eddy covariance at submesoscales. In the results presented below, only the eddy component is used to represent the submesoscale vertical flux, unless otherwise specified.

The filter is not a Reynolds operator: it is neither idempotent nor guaranteed to eliminate residual; that is, $\bar{\bar{\phi}} \neq \bar{\phi}$ and $\bar{\phi}' \neq 0$. Consequently, the residual flux is composed of three terms; for the finescale vertical tracer flux F_v' , this can be expressed as

$$F_v' = \overline{w\bar{c}} - \bar{w}\bar{c} = \underbrace{\overline{w\bar{c}} - \bar{w}\bar{c}}_{\text{Leonard}} + \underbrace{\overline{w\bar{c}'} + \overline{w'\bar{c}}}_{\text{Clark}} + \underbrace{\overline{w'\bar{c}'}}_{\text{Reynolds}}, \quad (18)$$

where the Leonard, Clark, and Reynolds terms represent filtered–filtered, filtered–subfilter, and subfilter–subfilter interactions, respectively. The Leonard and Clark terms are nontrivial here, but their summed contributions are substantially smaller than the Reynolds term on average (not shown). For coarse-resolution models that can only resolve down to the filter scale ℓ , it is the sum of all three terms that needs to be parameterized. In the following, finescale flux refers to the sum of all three terms in Equation 18, except in Section 4.3.2, where only the Reynolds term is analyzed.

4. Results

4.1. General Description

Despite the imposed surface cooling, MLIs continue to develop into eddies that are surrounded by strong meandering fronts with lateral scale of $\mathcal{O}(100)$ m, or arms forming a “λ”-shaped structure. The surface-fluxed tracer also exhibits rich spatial structures. This section qualitatively describes the 3D flow features and tracer distributions using representative late-time (hour 41) snapshots (Figure 3) from the forced simulations. A smaller subdomain and cross-front transects from the weak-cooling simulation are also shown in Figure 4. The no-cooling simulation has largely similar patterns, except that in the quiescent, non-frontal regions the tracer only accumulates at the surface. Overall, stronger surface-forced turbulence in the moderate-cooling case leads to more homogeneous tracer and vertical velocity distributions in both the vertical and horizontal directions. Below we contrast the characteristics of subdomains with and without fronts. In each simulation, a broad frontal zone is defined as regions where the magnitude of $\nabla_h \bar{b}$, averaged over the upper 60 m, exceeds $1 \times 10^{-8} \text{ s}^{-2}$, noting that 60 m is approximately the vertical scale (H_0) of the initial density fronts. The description focuses primarily on the weak-cooling simulation, but the features noted are also present in the moderate-cooling case, albeit less pronounced.

Away from the frontal zone, vertical velocity is dominated by surface-initiated plumes with characteristic velocity scale for convection, $w_* = (B_0 H)^{1/3}$. The tracer is transported predominantly downward while remaining relatively uniform in the horizontal, although variations imprinted by convective cells are visible at scales comparable to the local ML depth (30–60 m). Because our definition of ML is slightly deeper than the turbulent boundary layer (see Section 2), almost all tracer is confined in the ML. These results follow, as expected, canonical understanding of tracers in homogeneous convective turbulence (Mensa et al., 2015).

Inside the frontal zone, spatial patterns of tracer and flow are more complicated. Broadly speaking, there are two regimes near the surface, each occupying a specific part of the frontal zone. The first one is located inside eddies and in regions that have already restratified, where convective turbulence acts on the background state shaped by MLI (e.g., Figures 4a and 4b). In these areas the tracer and vertical velocity fields are roughly similar to those outside the frontal zone. Subtle differences arise in the morphology of turbulent plumes, which are often strained and distorted by small-scale horizontal flows, and occasionally organize into banded patterns (e.g., near $x = -2,800$ m and $y = 2,800$ m in Figures 4a and 4b). These subtleties in turbulence structure are later shown to also affect the transport efficiency of tracer (Section 4.3.2).

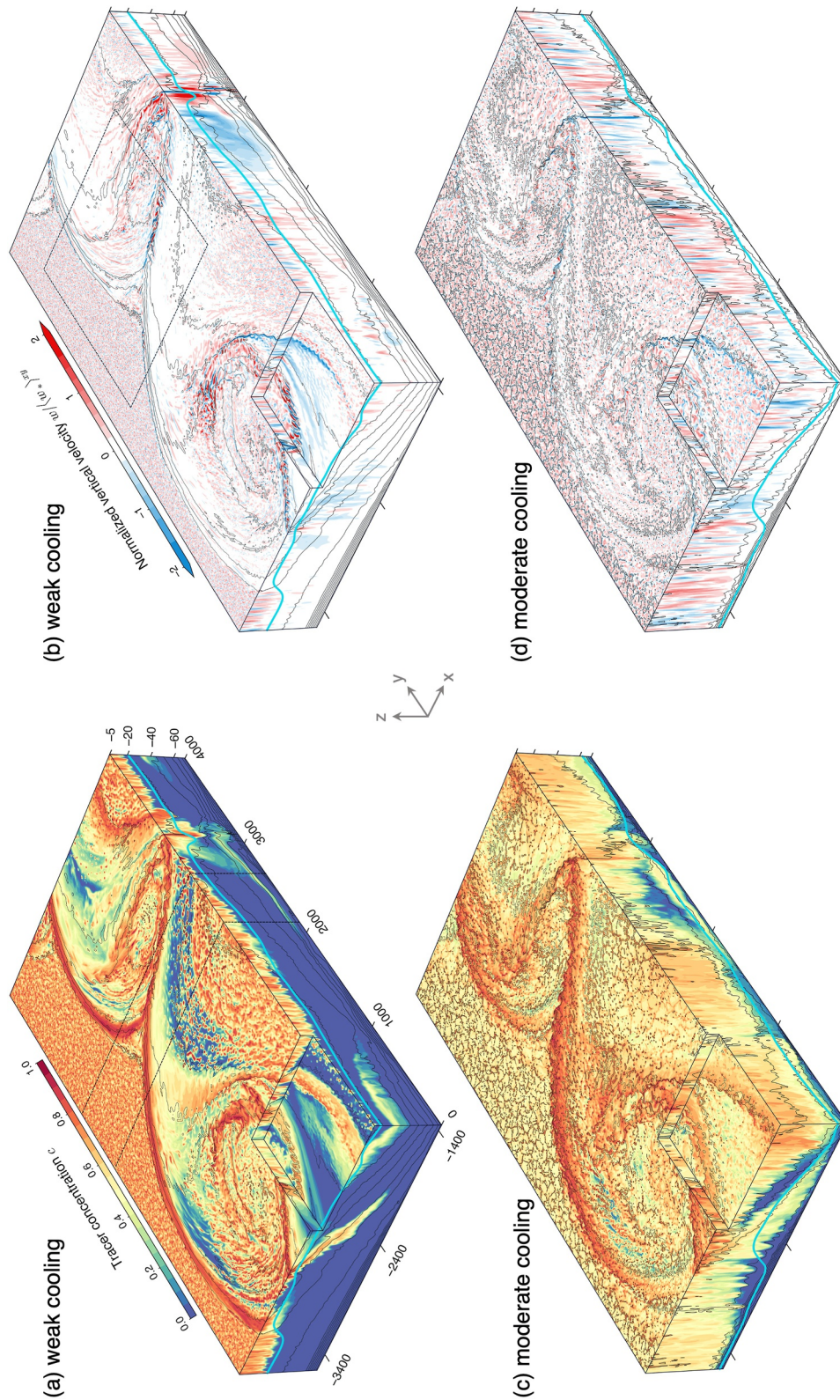


Figure 3. Three-dimensional snapshots of tracer concentration c and vertical velocity w normalized by the domain-averaged convective velocity scale $(w_e)^{1/2}$ at the left front of the simulation with (a and b) weak cooling and (c and d) moderate cooling at hour 41. Spatial coordinates have units of meters. The foreground cutout is approximately $900 \times 900 \text{ m}$ wide and 10 m deep. Dashed lines in panel (a) indicate the location of the slices in Figures 4c and 4d, which also intersect the dashed box frame in (b) that marks the location of the subdomain in Figures 4a and 4b. Cyan lines show the instantaneous mixed-layer depth. Black contours show the buoyancy b with interval $\Delta b = 1 \times 10^{-5} \text{ m s}^{-2}$.

The second regime is mostly active around sharp fronts. Following the meandering arms, the tracer is highly concentrated due to shallow ML and frontogenetic horizontal advection. This submesoscale frontogenesis triggers small-scale shear instability to develop along the arms, generating strong vertical velocity that does not scale with w_* (see the cutout region in Figure 3). Shear instabilities at the arms are a mixture of billows in the horizontal and vertical directions (Figures 3a, 3b, and 4d). These are shown in detail later in Section 4.5 by the frontogenetic tendency and KE productions. The resulting small-scale and intense vertical motions are often embedded within the downwelling part of the submesoscale overturning circulation. Similar feature of vertical velocity at sharp fronts have been reported in previous turbulence-resolving simulations of MLI (Stamper & Taylor, 2016; Taylor, 2016; Taylor et al., 2020). Here, vertical velocity is also elevated at highly curved fronts, around the narrow end of the emerging elliptical eddy (e.g., near $x = -1,400$ m and $y = 3,200$ m in Figure 3). This curvature-induced vertical motion could be understood using the cyclogeostrophic balance, which suggests that the flow should slow down as curvature increases and speed up as curvature reduces, thereby creating horizontal convergence and divergence as the flow enters and exits the cyclonically curved corner. This provides an effective mechanism for along-isopycnal transport, as highlighted in recent work investigating cyclone splitting and subsequent subduction of chlorophyll-rich surface waters observed in the Mediterranean (Middleton et al., 2025; Wu et al., 2025). However, our high-resolution simulations indicate this submesoscale flow curvature-induced horizontal convergence (divergence) is also accompanied by small-scale shear instability and internal wave excitation, both of which could contribute to diapycnal fluxes.

Finally, at greater depths, tracer-rich water is subducted out from the ML nearly along slanted isopycnals that outcrop at surface meandering arms (Figures 3a, 3c, 4c, and 4d). Depending on the location within the frontal zone, some of the subducted tracer filaments are evidently not solely the result of submesoscale along-isopycnal transport, because the tracer-weighted density typically shifts from lighter to denser classes during descent. Near sharp fronts, tracer subduction is usually accompanied by intense shear instabilities and potential diapycnal mixing (see $x = -2,000$ to $-1,700$ m in Figure 4d). This diapycnal contribution may be important, as it determines the density class in which the subducted tracer ultimately settles, and the pathways along which it would be dispersed by along-isopycnal motions of mesoscale eddies and large-scale circulation (Section 5).

4.2. Bulk Tracer Evolution and Mean Flux Profiles

With surface cooling, tracer is transported downward both inside and outside the frontal zone. However, the vertical structure and temporal evolution is very different in those two regions. Outside the frontal zone, the ML deepens as the convective plumes erode stratification, and the tracer transport is done by finescale flux alone. As the finescale tracer flux terminates above the ML depth (Figures 5c and 5f), the tracer does not penetrate below the ML (Figures 5a and 5d). The mean tracer concentration, finescale buoyancy and tracer flux (gray lines in Figure 5) all exhibit universal shapes, as expected from the Monin and Obukhov (1954) similarity theory for horizontally homogeneous boundary layers. Here the surface flux is negatively proportional to the surface concentration, therefore is stronger at earlier times, and gradually decreases to a smaller rate as the surface concentration approaches saturation. As a result, the time-mean finescale tracer flux does not decay linearly from the surface with a constant slope, as would be the case for a tracer forced with a steady surface flux (Chor et al., 2020).

Averaged over the entire frontal zone, mean buoyancy profiles have a stronger vertical gradient than those outside (not shown), and the finescale buoyancy flux no longer conforms to a linear decay from the surface (orange lines in Figures 5b and 5e). The curvature of the finescale buoyancy flux profile is affected by the presence of submesoscale buoyancy flux; together, their vertical gradients determine whether the near-surface water column experiences destratification or restratification. The mean tracer concentration profiles (Figures 5a and 5d) are also less well-mixed than those outside the frontal zone. They typically extend below the ML (indicating mean subduction) and progressively penetrate deeper over time, even though the ML depth remains relatively steady. This behavior is usually attributed to submesoscale flux whose vertical scale is set by the front, rather than by the shoaled ML depth (Lévy et al., 2018). The fronts in our simulations reach deeper than the ML, so submesoscale flow can readily carry tracer to the interior. In contrast to that outside the front, the mean finescale tracer flux remains significant below the ML (Figures 5c and 5f). Its vertical extent exceeds the ML depth but appears to be bounded by that of the front. This deep-reaching finescale transport of passive tracer is not found for buoyancy (Figures 5b and 5e). Although there is a small amount of finescale buoyancy flux below the ML in the weak-cooling case, it is likely associated with the conversion of available PE by MLI. This is evident from

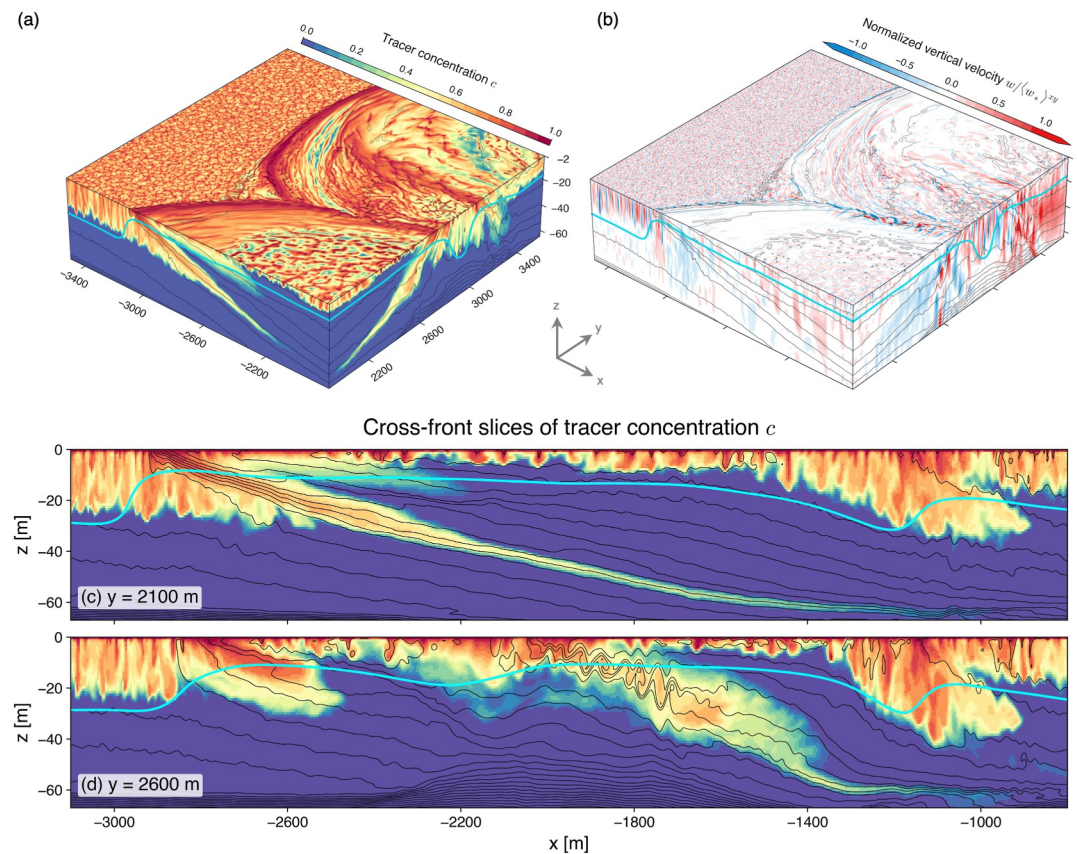


Figure 4. Three-dimensional snapshots of (a) tracer concentration c and (b) vertical velocity w normalized by the domain-averaged convective velocity scale $(w_*)^{xy}$ for a smaller subdomain (dashed box frame in Figure 3b) from the weak-cooling simulation at hour 41. (c and d) Vertical transects of cross-frontal distribution of tracer concentration c at different y locations (dashed lines in Figure 3a) from the weak-cooling simulation at hour 41. Spatial coordinates in panel (a and b) have units of meters and the top slices are from a shallower depth (2 m) than those in Figure 3. Cyan lines show the instantaneous mixed-layer depth. Black contours show the buoyancy b with interval $\Delta b = 1 \times 10^{-5} \text{ m s}^{-2}$ in panel (a and b) and $5 \times 10^{-6} \text{ m s}^{-2}$ in (c and d).

comparing the finescale part of buoyancy flux in the forced run to the unforced run (orange dashed lines in Figure 5b). This effect becomes negligible in the moderate-cooling case (Figure 5e), which noting that the separation scale in the weak-cooling case is slightly smaller than ℓ (Figures 2c and 2d), suggests that it is probably a consequence of slight aliasing of the submesoscale due to the fixed filter scale ℓ . This scale contamination is not a major contributor to the deep finescale tracer flux in the weak-cooling case, as it is much larger than that in the unforced run. A decomposition of the finescale vertical tracer flux into diapycnal and isopycnal components (Section 5) also supports this argument.

Overall, the bulk evolution and mean fluxes of these two forced simulations are very similar. With stronger cooling, the ML depth is deeper and the vertical gradient of mean tracer concentration is smaller. Similar to previous studies (Hamlington et al., 2014; Verma et al., 2022), the submesoscale buoyancy flux becomes stronger as the surface forcing increases. This is primarily due to the increase of submesoscale available PE reservoir through vertical mixing of buoyancy. This variation of submesoscale buoyancy flux with forcing is mostly limited in the near-surface layer, where the vertical mixing of buoyancy is active. In the frontal zone, the finescale tracer flux is enhanced by surface cooling, and the submesoscale tracer flux is also about 50% higher in the moderate-cooling case. This moderate increase of submesoscale transport may be attributed to enhanced submesoscale energetics (stronger advection and diffusion), as well as to the modulation of tracer gradient by finescale turbulence—that is, more tracer available for transport by the submesoscale flow under stronger surface forcing conditions. The relative importance of these effects is beyond the scope of this study and is left for future work.

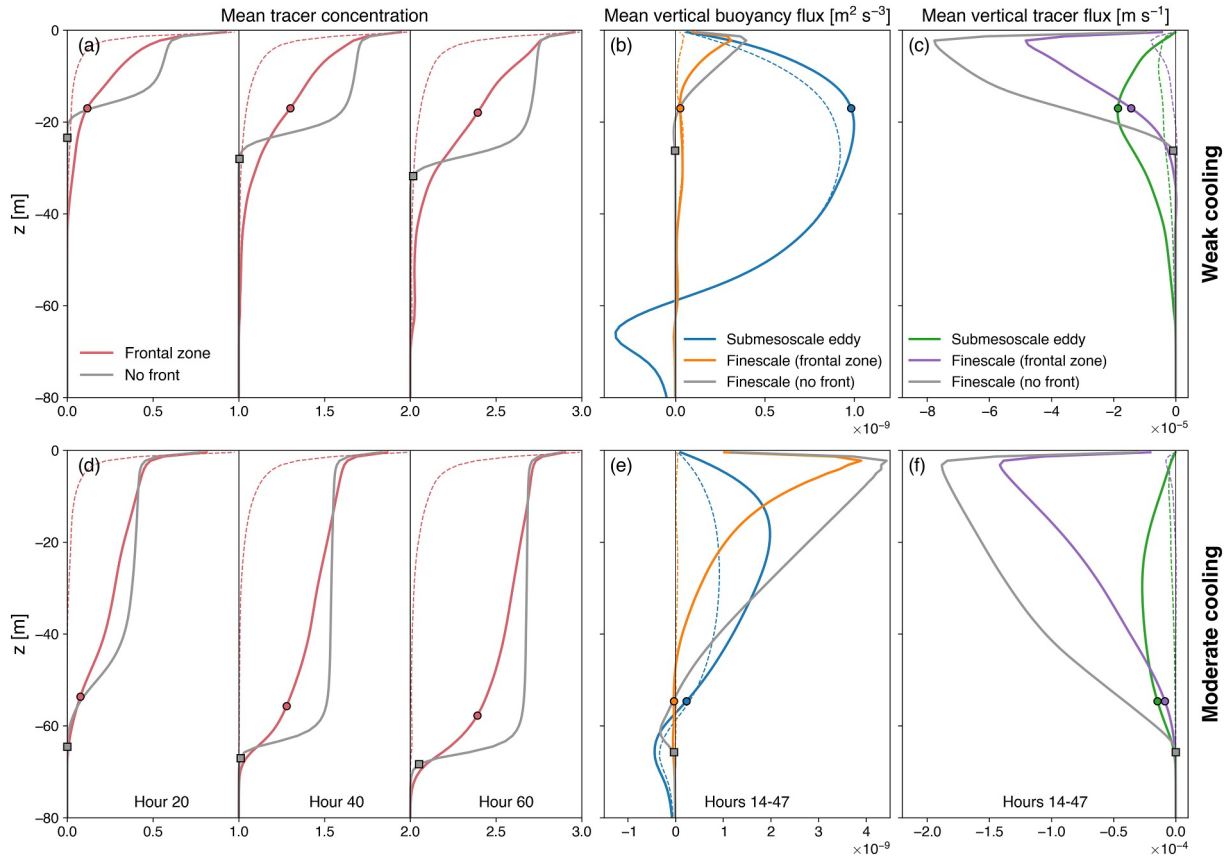


Figure 5. Profiles of mean tracer concentration, mean vertical buoyancy fluxes, and mean vertical tracer fluxes for simulation forced with (a–c) weak and (d–f) strong surface cooling. Tracer concentrations are horizontally averaged inside and outside frontal zone (regions with large horizontal buoyancy gradient, see Section 4.1) at three different hours, and profiles from hour 40 and hour 60 are shifted by 1. Horizontal mean fluxes are further averaged over approximately two inertial periods (hours 14–47). Dashed lines in all subpanels are corresponding frontal zone profiles from the no-cooling simulation. Dots and squares show the mean mixed-layer depth inside and outside the frontal zone, respectively. Note different scales for horizontal axes in panel (b and e) and (c and f). Only the eddy component (see Equation 17) is shown for the submesoscale flux.

4.3. Conditional Averaging and Spatial Organization of Transport

In this subsection, we use conditional averaging and spatial maps of vertical tracer fluxes to better understand what patterns give rise to the mean transport shown in Section 4.2. We first examine the spatial characteristics of submesoscale and finescale fluxes across different submesoscale flow features, and then compare the turbulence structures responsible for the finescale flux inside and outside the frontal zone.

4.3.1. Vertical Tracer Fluxes Conditioned on Vorticity and Strain

Here, we follow Balwada et al. (2021) to attribute horizontal divergence δ and vertical tracer transport to different flow features in the parameter space of vertical relative vorticity and horizontal strain rate (ζ , σ). Even though our simulations have much finer resolution—so that horizontal flows are dominant only at the largest scales—this construct is still useful as it connects these simulations to submesoscale-permitting ones with parameterized finescale turbulence. The relevant flow kinematics are defined using filtered velocity gradients, thereby representing the horizontal divergence (hereafter “divergence”), vertical relative vorticity (hereafter “vorticity”), and horizontal strain rate (hereafter “strain,” including both normal and shear contributions) of the submesoscale flow—not of the total flow including turbulence,

$$\delta = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}, \quad \zeta = \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y}, \quad \sigma = \sqrt{\sigma_n^2 + \sigma_s^2} = \sqrt{\left(\frac{\partial \bar{u}}{\partial x} - \frac{\partial \bar{v}}{\partial y}\right)^2 + \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y}\right)^2}, \quad (19)$$

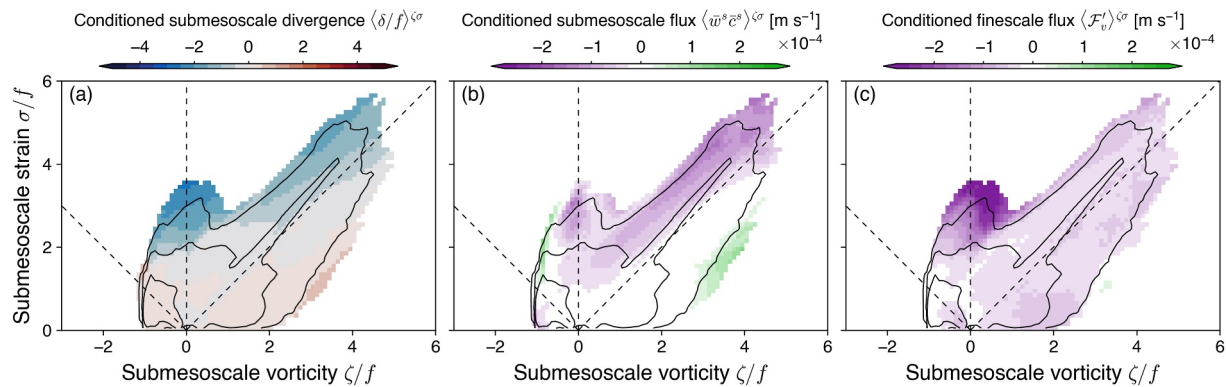


Figure 6. Conditional mean of (a) surface submesoscale divergence, (b) submesoscale tracer flux, and (c) finescale tracer flux based on surface submesoscale vorticity and strain in the weak-cooling simulation. Fluxes are evaluated at 10 m depth, inside the frontal zone. Data used for averaging span approximately two inertial periods (hours 14–47). Bins with less than 170 data counts are omitted. Contours show the highest 60%, 90%, and 99% centered distribution of vorticity and strain, respectively.

where σ_n and σ_s denote the normal and shear strain, respectively. The vorticity and strain provide a simple diagnostic that partitions the submesoscale flow field into regions dominated by fronts ($\sigma > |\zeta|$) and eddies ($\sigma < |\zeta|$). Consistent with Lagrangian analysis results (Freilich & Mahadevan, 2021), conditional averaging of RANS simulations in the vorticity–strain space (Balwada et al., 2021; Picard et al., 2024) have shown that strain-dominated submesoscale fronts play an outsized role in vertical tracer subduction, despite their diminutive spatial footprint. However, it remains unclear whether this spatial organization of the submesoscale flux holds in turbulence-resolving LES, and whether the finescale flux exhibits a similar organization. To address these questions, we evaluate the conditional averages of both submesoscale and finescale fluxes at depth in the normalized submesoscale vorticity–strain space. Surface values of vorticity and strain are used, in part to maintain consistency with earlier work, but also because they are directly relevant to observations, as surface flow properties are more readily measured from remote sensing (Farrar et al., 2025). Because our simulations lack a complex mélange of flow features at scales larger than the submesoscale, the area fraction of fronts is not representative of the real ocean. Consequently, we do not attempt to attribute the transport in an area-weighted manner and instead restrict this analysis to the frontal zone.

Figure 6 shows the conditionally averaged surface divergence and vertical tracer fluxes using data spanning approximately two inertial periods (hours 14–47) from the weak-cooling simulation. Two groups of snapshots (hours 14 and 31) are shown in Figure 7 to illustrate the spatial structures discussed below. Results from the moderate-cooling simulation are similar. Fluxes are evaluated at 10 m depth, approximately half of the mean ML depth in the frontal zone (see Figure 5c). Despite the highly idealized frontal spindown setup with only a few eddies, the vorticity–strain joint probability density function (JPDF) displays a shape broadly consistent with prior observational results (Shcherbina et al., 2013) and RANS simulations (Balwada et al., 2021). The JPDF from LES also shows some subtle differences, including a sharp cutoff around $\zeta/f = -1$, and a secondary bump in the region with low vorticity and high strain (LVHS). Spatially, the LVHS region corresponds to the secondary arm formed on the warm side of the initial front (see dashed ellipses in Figure 7). The conditioned surface divergence exhibits a pattern nearly identical to that in RANS simulations, with surface convergence concentrated around strain-dominated fronts. The only difference is the absence of divergence signatures from anticyclonic eddies, which do not occur in our simulations. The conditioned submesoscale eddy flux $\langle \bar{w}^s \bar{c}^s \rangle^{\zeta\sigma}$ indicates that strong downward transport is concentrated on the cyclonic side of fronts and within the LVHS region, weaker downward transport in areas dominated by anticyclonic vorticity. Upward transport is generally weaker and occurs on the anticyclonic side of fronts and around cyclonic eddies.

For finescale tracer flux, the same conditional averaging highlights the importance of the LVHS region among other regions (including cyclonic eddies) of downward transport, especially at early times of the MLI development (Figure 7e). This partition in vorticity–strain space is consistent with the qualitative description of frontal zone structures in Section 4.1. The conditional averaging of KPP-parameterized finescale tracer flux in Balwada et al. (2021) also shows enhanced transport with increasing strain in low-vorticity areas (their Figure 9d), even though the RANS simulation does not exhibit an LVHS bump in the JPDF. However, the finescale flux (evaluated

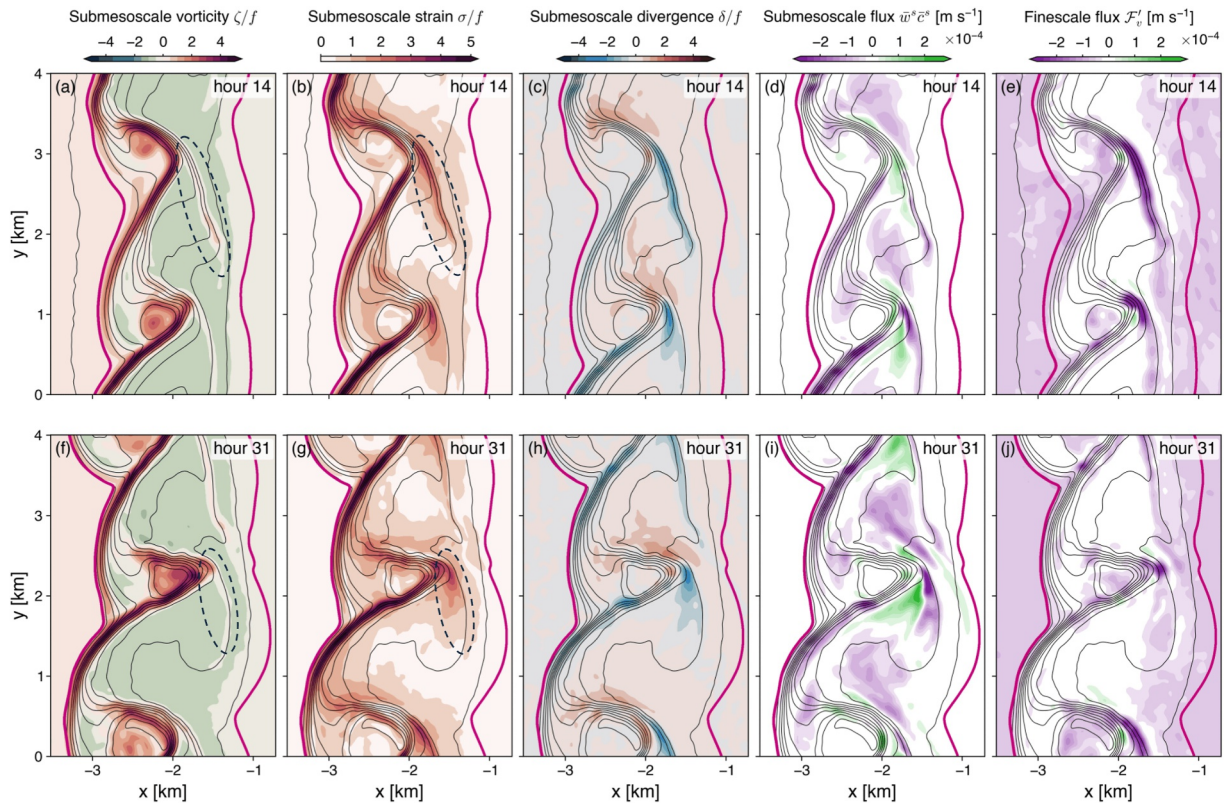


Figure 7. Snapshots of surface submesoscale vorticity, surface submesoscale strain, surface submesoscale divergence, submesoscale vertical tracer flux, and finescale vertical tracer flux at (a–e) hour 14 and (f–j) hour 31 from the weak-cooling simulation. Fluxes are evaluated at 10 m depth, approximately half of the mean mixed-layer depth in the frontal zone. Contours show surface buoyancy $\bar{b}$ with interval $\Delta \bar{b} = 5 \times 10^{-6} \text{ m s}^{-2}$. The frontal zone is encompassed by the two magenta lines. Examples of the low-vorticity high-strain (LVHS) region are shown by dashed ellipses in panel (a, b, f, and g).

at half ML depth) in our case is substantially larger—comparable in magnitude to the submesoscale flux at the same depth—whereas in Balwada et al. (2021), it is orders of magnitude smaller than the submesoscale component at the ML base.

These patterns of submesoscale and finescale vertical tracer transport, which largely persist over time (Figure 7), confirm previous findings of strain-dominated subduction in RANS simulations, but also reveals more nuanced spatial characteristics as additional flow features are resolved in high-resolution simulations.

4.3.2. Quadrant Analysis of Finescale Vertical Tracer Fluxes

The conditional averaging in the previous subsection is useful for examining the spatial structures of transport by submesoscale and finescale flows in the frontal zone, but is not suitable for comparing finescale turbulence structures between regions with and without fronts, since vorticity and strain are negligible outside the front. To facilitate this comparison, we turn to quadrant analysis (Wallace, 2016), which is commonly employed within the framework of Reynolds averaging to understand the connection between covariances (e.g., $\langle w'c' \rangle$, averaged product of fluctuation w' and c') and spatial structures of turbulent flow (e.g., streaks, cells). The products of these fluctuations can be categorized into four quadrants based on their sign combinations: Q1 ($w' > 0, c' > 0$), Q2 ($w' > 0, c' < 0$), Q3 ($w' < 0, c' < 0$), Q4 ($w' < 0, c' > 0$). Each quadrant is related to a specific type of motion that contributes to the total covariance (or flux). For the turbulent flux of a surface-fluxed tracer $\langle w'c' \rangle$, Q2 (“sweeps”) corresponds to upwelling of low-concentration water, and Q4 (“ejections”) corresponds to downwelling of high-concentration water; both contribute positively to the total flux. Q1 and Q3, while contributing negatively to the total flux, can be interpreted in the same manner. The contribution of each quadrant can be determined by conditional averaging of the quadrant-sorted data. Meanwhile, one can also define a transport efficiency $\eta_{w'c'}$ (Wyngaard & Moeng, 1992) based on the ratio of the total flux to the flux from positively contributing quadrants; in this case,

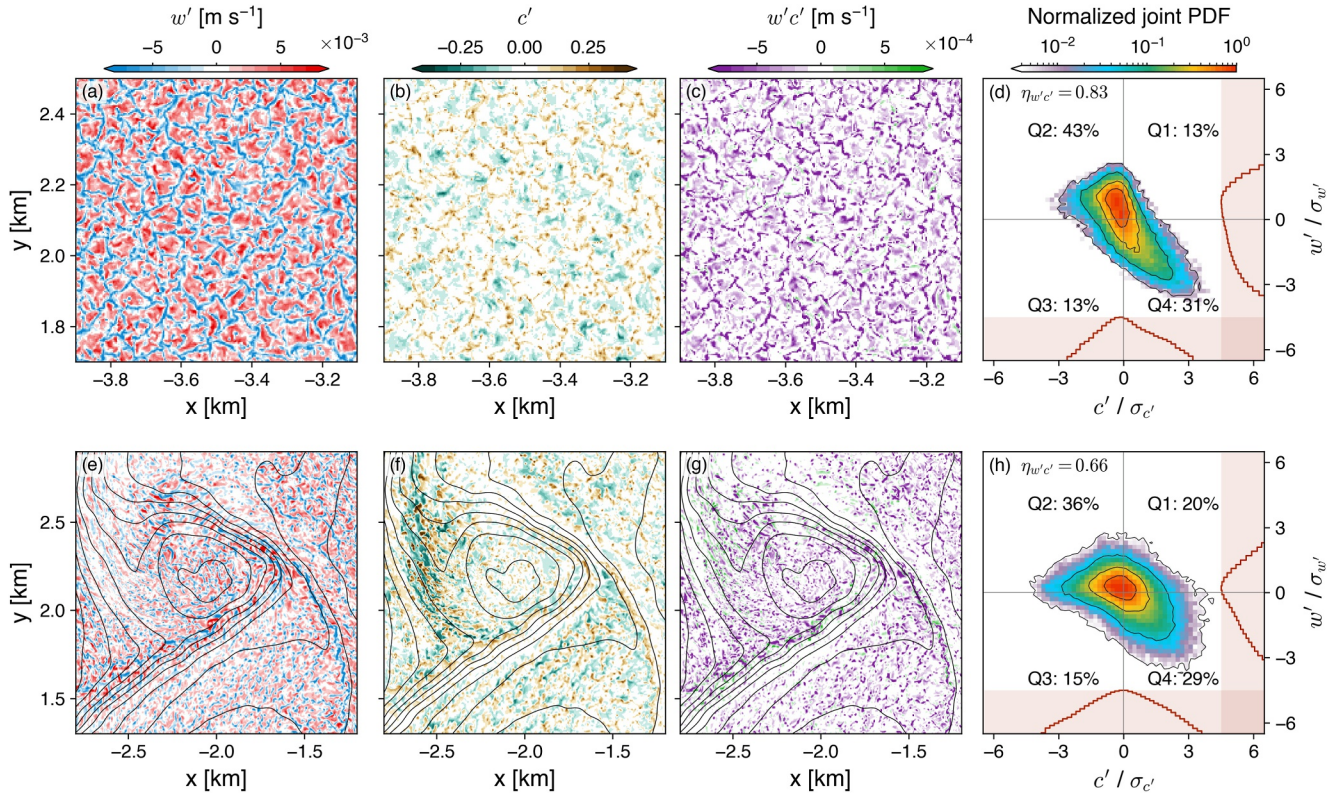


Figure 8. Near-surface (10 m depth) finescale vertical velocity w' , tracer concentration c' , the product of finescale anomalies $w'c'$, and the associated joint probability density function for regions (a–d) outside and (e–h) inside the frontal zone. Data taken from the simulation with moderate cooling at hour 31. Black contours in panel (e–g) show the buoyancy $\bar{b}$ at the same depth with interval $\Delta \bar{b} = 7 \times 10^{-6} \text{ m s}^{-2}$. In panel (d and h), axes are normalized by the same standard deviation; contours show the highest 30%, 60%, 90%, and 99% centered distribution, respectively; stair plots on the side show the shape of normalized 1D PDF on logarithmic scale from 10^{-2} to 10^0 over the red shading background. Note the horizontal extent of the slices in the bottom row is twice of those in the top row.

$$\eta_{w'c'} = \frac{\langle w'c' \rangle}{\langle w'c' \rangle_{Q2} + \langle w'c' \rangle_{Q4}}, \quad (20)$$

where the superscript on the averaging operator $\langle \rangle$ denotes the contribution from a given quadrant, and the numerator is the sum over all quadrants. The transport efficiency ranges from 0 to 1, and is linked to the correlation coefficient between w' and c' . This quantity, together with the JPDF of w' and c' , has important utility in mass-flux closures that parameterize transport by coherent turbulence structures (Arakawa, 1969).

Although the finescale perturbations here are not obtained through Reynolds averaging, quadrant analysis can still provide insight into the difference of turbulence structures inside and outside the frontal zone, as the Reynolds component of the finescale flux $\mathcal{F}'_v$ usually dominates. Figure 8 compares the near-surface (10 m depth) JPDF of w' and c' in frontal and non-frontal regions along with their horizontal slices, using data from the moderate-cooling simulation. Outside the frontal zone, both w' and c' bear the signature of convective cells, where high-concentration water sinks down at the cell edge, and low-concentration water rises up in the cell center. The product $w'c'$ is negative almost everywhere but the largest magnitude tends to cluster around cell edges where w' and c' are highly correlated. This spatial structure leads to the dominance of Q2 and Q4 in the JPDF of $w'c'$ (43% and 31%). The stronger flux at the cell edges manifests as a long tail in Q4 (Figure 8d), primarily due to the strongly skewed 1D distribution of w' . The overall shape of the JPDF with a transport efficiency around 0.83 is typical of horizontally homogeneous convective boundary layers (Chowdhuri & Prabha, 2018).

Within the frontal zone, slices of w' and c' show a more heterogeneous picture, except inside the cyclonic eddies. The finescale vertical velocity usually has higher magnitude along sharp fronts due to shear instability

(Section 4.1), while the finescale tracer anomaly has higher magnitude in regions that have restratified (shallow ML). Moreover, the spatial patterns of w' and c' exhibit little similarity, and the product $w'c'$ has both positive and negative values, both at sharp fronts and in restratified regions. The JPFD in the frontal zone shows a more Gaussian-like shape, especially for the highest 90% distribution. As a result, the transport efficiency (about 0.66) for vertical tracer flux is much lower in frontal regions even though the vertical velocity variance remains high around sharp fronts. This explains why the near-surface mean finescale tracer flux in the frontal zone is weaker than that outside the front (see Figures 6c and 6f). This inefficient transport of tracer and heat (not shown) is potentially related to the presence of shear instabilities. First, significant horizontal velocity fluctuations from horizontal shear instability generate finescale tracer anomalies that do not correlate with w' . Second, shear instability shifts the dominant turbulent coherent structure away from convective plumes that are efficient for heat (tracer) transport (D. Li & Bou-Zeid, 2011). This difference in turbulence structure and transport efficiency may be analogous to the cell-to-roll transition with increasing vertical shear (Salesky et al., 2017), and the stability-dependent turbulent Prandtl number (D. Li, 2019) observed in atmospheric boundary layers.

4.4. Budget for Tracer Below Mixed-Layer Depth

For a bulk quantification of the tracer exchange between ML and interior, we integrate the filtered tracer concentration $\bar{c}$ from a depth near the bottom ($z_b \approx -120$ m) to the ML base [$z_m = -H(x, y, t)$], and over the entire horizontal domain. We call this integrated quantity the interior tracer, $\bar{C} = \iint_A \int_{z_b}^{z_m} \bar{c} dz dx dy$ (the horizontal integral bounds are omitted). Using the Leibniz rule and horizontal periodicity, the budget for the interior tracer $\bar{C}$ is,

$$\frac{\partial \bar{C}}{\partial t} = \iint_A \left[\underbrace{-\frac{\partial H}{\partial t} \bar{c}|_{z_m}}_{\text{restratification}} + \underbrace{(\bar{\mathcal{F}}_h + \mathcal{F}'_h)|_{z_m} \cdot (-\nabla_h H)}_{\text{lateral}} + \underbrace{(-\bar{\mathcal{F}}_v|_{z_m})}_{\text{submesoscale}} + \underbrace{(-\mathcal{F}'_v|_{z_m})}_{\text{finescale}} \right] dx dy, \quad (21)$$

where the first two terms on the right-hand side represent the contributions from restratification (or destratification) and lateral transport associated with time- and space-varying ML depth; the last two terms represent vertical transport across the ML base by submesoscale and finescale motions, respectively. Note that here the total transport from lateral and vertical fluxes is exactly equal to the integral of the flux perpendicular to the ML base over the tilted H surface, consistent with the divergence theorem.

We diagnose the budget terms for $\bar{C}$ by vertically interpolating hourly fields to the evolving ML depth. Snapshots of vertical mean tracer concentration in the ML and interior from the weak-cooling simulation are shown in Figures 9a and 9b. The ML mean tracer concentration is elevated around the “ λ ”-shaped arms. The interior tracer is present only within the frontal zone and is likewise concentrated along these arms, but displays a broader distribution due to cross-frontal advection and diffusion. Exchange between the two tracer reservoirs occurs through the terms on the right-hand side of Equation 21; their spatial maps are shown in Figures 9d–9h along with the ML depth distribution (Figure 9c). The first term leads to more interior tracer if the ML shoals ($\partial H/\partial t < 0$, restratification), and less interior tracer if the ML deepens ($\partial H/\partial t > 0$, destratification). In our simulations, this term is dominated by restratification in the frontal zone (Figure 9d) because the restratification is much faster, and the tracer concentration at ML base is also higher. The lateral transport is only important where the ML base varies significantly in space, and is often dominated by submesoscale horizontal flux (Figure 9e). The finescale counterpart $\mathcal{F}'_h|_{z_m}$ is much smaller (Figure 9f). The contribution from restratification and lateral transport are highly correlated in space but typically have opposite signs. As expected, the submesoscale vertical transport across the ML depth (Figure 9g) is broadly active in the frontal zone, and is a major contributor to tracer subduction. Notably, the finescale vertical flux (Figure 9h) remains important at the ML base, but only near regions with sharp fronts, occupying approximately 7% of the frontal zone area. This is consistent with the nonzero mean finescale flux at the mean ML depth shown in Figure 5; however, the role of this finescale vertical flux is more precisely quantified here by accounting for spatial variations of the ML depth.

The time series of cumulative contributions from restratification, lateral transport, and vertical transport are shown in Figure 10 for both simulations. The lateral transport includes both submesoscale and finescale horizontal fluxes

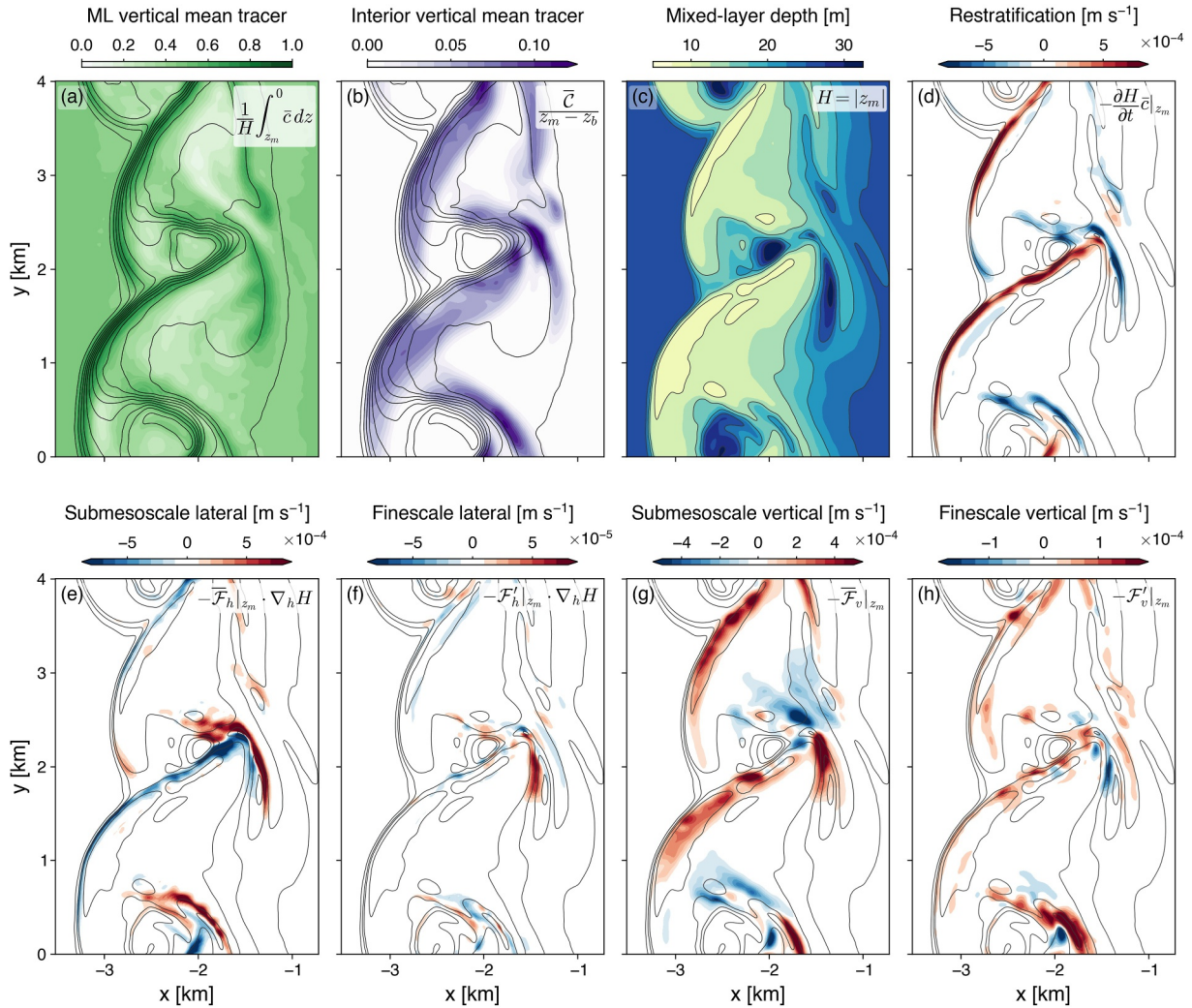


Figure 9. Spatial maps of (a) mixed layer (ML) vertical mean tracer concentration, (b) interior vertical mean tracer concentration, (c) mixed-layer depth, (d–h) budget terms for the interior tracer $\bar{C}$ in Equation 21, at hour 31 of the weak-cooling simulation. Contours in panel (a and b) show the surface buoyancy $\bar{b}$ with interval $\Delta \bar{b} = 5 \times 10^{-6} \text{ m s}^{-2}$. Contours in panel (c–h) show the ML depth distribution at 5 m interval.

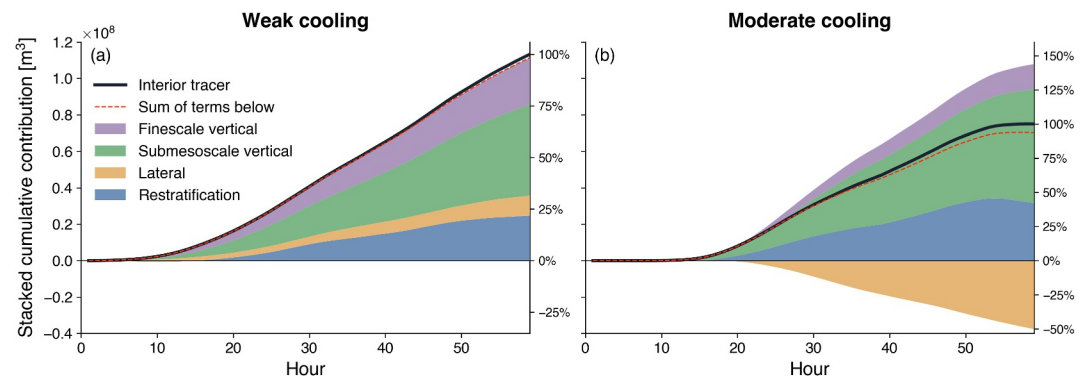


Figure 10. Time-integrated budget of the interior tracer $\bar{C}$ according to Equation 21 for the (a) weak-cooling and (b) moderate-cooling simulation. Small residual gaps are mostly due to accumulation of numerical errors from calculations using hourly data.

but is dominated by the former. After 60 hr of simulation with weak surface cooling, submesoscale vertical flux accounts for nearly half (~45%) of the total tracer below the ML depth. Restratification and lateral transport together contribute about 30%, though their effects diminish toward the end of the simulation as variations of ML depth decrease with time. Interestingly, finescale vertical flux provides a comparable contribution (~25%) to the total subduction, and, like submesoscale vertical flux, persists in time. In the moderate-cooling case, submesoscale vertical flux remains the dominate source for the interior tracer. Lateral transport, however, moves subducted tracer back into the ML, roughly offsetting the effect of restratification. This difference occurs mainly in regions with strong flow curvature, where stronger cooling increases both the horizontal scale of the meandering currents and the horizontal gradient of the ML depth. Finescale vertical flux, though smaller, still contributes about 15% of the total subduction. This tracer budget analysis suggests that both submesoscale and finescale vertical fluxes are important direct contributors to tracer subduction in the frontal zone. Restratification and lateral transport can also lead to indirect subduction, though the combined effects depend strongly on the evolution of local ML depth.

4.5. Localized Finescale Tracer Subduction Linked to Frontogenesis

The enhanced finescale tracer flux near the ML base is highly localized near sharp frontal regions (Figure 9h); this implies a connection between finescale tracer subduction and frontogenesis—the processes by which an initially broad front is rapidly intensified through a positive feedback (Hoskins, 1982; Hoskins & Bretherton, 1972; McWilliams, 2021). The rate at which the front is sharpened can be quantified using the frontal tendency equation (see also Capet et al., 2008; Gula et al., 2014) derived from Equation 12,

$$\begin{aligned} \frac{\bar{D}|\nabla_h \bar{b}|^2/2}{\bar{D}t} = & - \underbrace{\left[\left(\frac{\partial \bar{b}}{\partial x} \right)^2 - \left(\frac{\partial \bar{b}}{\partial y} \right)^2 \right] \frac{\sigma_n}{2} - \frac{\partial \bar{b}}{\partial x} \frac{\partial \bar{b}}{\partial y} \sigma_s}_{\mathcal{T}_{h,\sigma}^b} + \underbrace{|\nabla_h \bar{b}|^2 \left(\frac{-\delta}{2} \right)}_{\mathcal{T}_{h,-\delta}^b} + \underbrace{\frac{\partial \bar{b}}{\partial z} (-\nabla_h \bar{w} \cdot \nabla_h \bar{b})}_{\mathcal{T}_v^b} \\ & + \underbrace{[\nabla_h (-\nabla \cdot \mathbf{B}')] \cdot \nabla_h \bar{b}}_{\text{finescale buoyancy mixing}}, \end{aligned} \quad (22)$$

where $\frac{\bar{D}}{\bar{D}t}$ is the Lagrangian rate of change following the filtered velocity $\bar{\mathbf{u}}$. Following Srinivasan et al. (2023), the total adiabatic frontogenetic tendency $\mathcal{T}^b$ is decomposed into three terms: one proportional to the strain $\mathcal{T}_{h,\sigma}^b$, one proportional to the convergence $\mathcal{T}_{h,-\delta}^b$, and one associated with the differential vertical advection $\mathcal{T}_v^b$.

The connection between finescale tracer subduction and frontogenesis can be understood in light of recent work showing that frontogenesis at submesoscale drives a forward KE flux toward smaller scales (Srinivasan et al., 2023). The resulting enhancement of finescale KE in frontogenetic regions then leads to strong finescale tracer flux by strengthening the vertical gradient production term $-\tau_{ww}(\partial \bar{c}/\partial z)$ in the budget for F'_v (note that $2\tau_{ww}$ is the vertical component of finescale KE). This connection is qualitatively supported by the spatial correspondence between the finescale flux F'_v (Figure 9h) and the finescale stress τ_{ww} at the ML base, as well as the near-surface frontogenetic tendency $\mathcal{T}^b$ (Figures 11a and 11c). However, frontogenesis is not the only source for finescale KE as the simulation is also forced by surface cooling. Next, we examine the finescale KE budget to assess whether the elevated finescale KE arises from localized convective organization at fronts, or from submesoscale processes that are only active in certain parts of the frontal zone.

In the framework of coarse-graining, the finescale KE is defined as

$$\mathcal{E}' = \underbrace{(\tau_{uu} + \tau_{vv})/2}_{\mathcal{E}'_h} + \underbrace{\tau_{ww}/2}_{\mathcal{E}'_v}, \quad (23)$$

where $\tau_{uu}, \tau_{vv}, \tau_{ww}$ are the diagonal element of the finescale stress tensor $\boldsymbol{\tau}'$ in Equation 16, and $\mathcal{E}'_h$ and $\mathcal{E}'_v$ denote the horizontal and vertical component of finescale KE, respectively. Following Germano (1992), the budget equation for finescale KE is

$$\frac{\partial \mathcal{E}'}{\partial t} + \nabla \cdot \mathbf{J}' = B'_v + \Pi_h + \Pi_v - \epsilon, \quad (24)$$

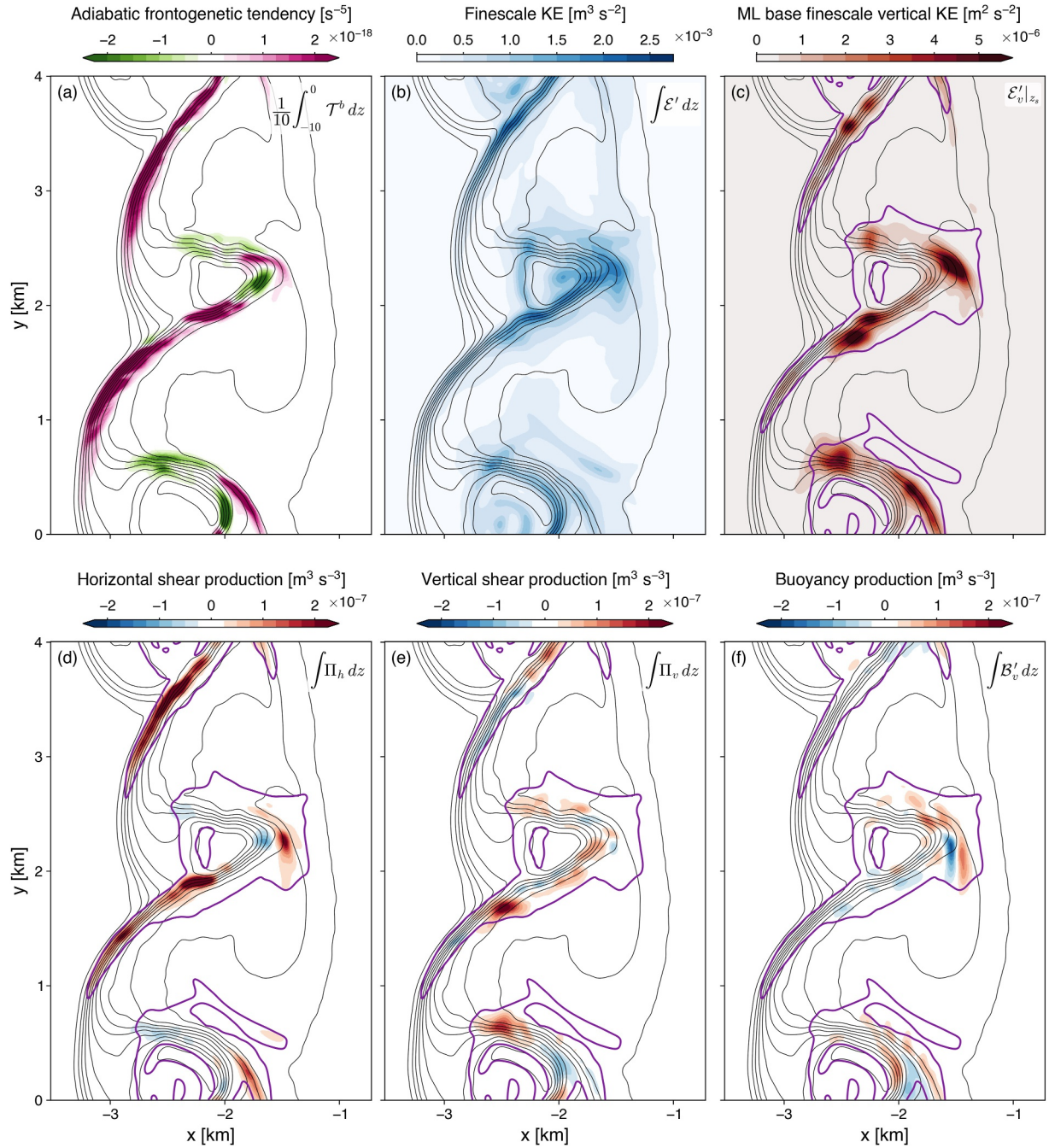


Figure 11. Spatial maps of (a) adiabatic frontogenetic tendency, (b) finescale kinetic energy (KE), (c) mixed-layer (ML) base finescale vertical KE, (d) horizontal shear production, (e) vertical shear production, and (f) vertical buoyancy flux at hour 31 from the weak-cooling simulation. The frontogenetic tendency is averaged over the upper 10 m; the finescale KE and energy production terms are vertically integrated from z_b (≈ -120 m) to surface. Black contours of surface buoyancy $\bar{b}$ indicate the location of surface fronts. Purple lines encircle sharp frontal regions, roughly identified with elevated finescale KE ($\int \mathcal{E}' dz > 5 \times 10^{-4} \text{ m}^3 \text{ s}^{-2}$).

where J' represents the spatial redistribution of $\mathcal{E}'$ by submesoscale advection, finescale pressure transport, finescale turbulent transport, and SGS transport; B'_v is the finescale vertical buoyancy flux (or buoyancy production); Π_h and Π_v are the horizontal and vertical component of the total shear production Π ; ϵ is the dissipation rate of finescale KE. The detailed form of each term is shown in Equation 25 of Germano (1992). The redistribution term is spatially complex at fronts (Bo et al., 2026; Sullivan & McWilliams, 2024), but it does not

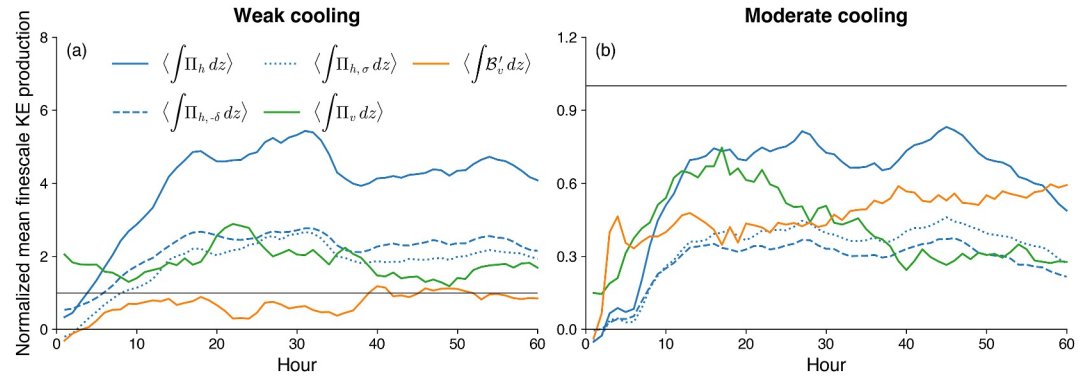


Figure 12. Time series of vertically integrated finescale kinetic energy production terms averaged in sharp frontal regions for the (a) weak-cooling and (b) moderate-cooling simulation. Production terms in each simulation are normalized by the corresponding spatial mean of buoyancy production (the only source of turbulence) in non-frontal regions at hour 60. Note the horizontal shear production is also decomposed into the convergence-driven ($\Pi_{h,-\delta}$) and the strain-driven ($\Pi_{h,\sigma}$) component.

contribute to integrated $\mathcal{E}'$; therefore, we focus on the production terms only. These are explicitly diagnosed as building on the definitions above for horizontal divergence and strain in Equation 19:

$$B'_v = \overline{wb} - \bar{w}\bar{b}, \quad (25)$$

$$\begin{aligned} \Pi_h &= - \begin{bmatrix} \tau_{uu} & \tau_{uw} \\ \tau_{uw} & \tau_{ww} \end{bmatrix} : \begin{bmatrix} \partial \bar{u}/\partial x & (\partial \bar{u}/\partial y + \partial \bar{v}/\partial x)/2 \\ (\partial \bar{u}/\partial y + \partial \bar{v}/\partial x)/2 & \partial \bar{v}/\partial y \end{bmatrix} \\ &= \underbrace{-(\tau_{uu} - \tau_{vv}) \frac{\sigma_n}{2} - \tau_{uw} \sigma_s}_{\Pi_{h,\sigma}} + \underbrace{(\tau_{uu} + \tau_{vv}) \left(\frac{-\delta}{2} \right)}_{\Pi_{h,-\delta}}, \end{aligned} \quad (26)$$

$$\Pi_v = - \begin{bmatrix} \tau_{uw} \\ \tau_{vw} \\ \tau_{ww} \end{bmatrix} : \begin{bmatrix} \partial \bar{u}/\partial z + \partial \bar{w}/\partial x \\ \partial \bar{v}/\partial z + \partial \bar{w}/\partial y \\ \partial \bar{w}/\partial z \end{bmatrix}, \quad (27)$$

where the operator “:” denotes the (doubly-contracted) inner product of two tensors. As in the frontal tendency equation above, we decompose the horizontal shear production Π_h into a strain-driven component $\Pi_{h,\sigma}$, and a convergence-driven component $\Pi_{h,-\delta}$. This decomposition highlights the connection between frontogenesis and cross-scale KE flux, and clearly shows the essential role of convergence and strain in both processes (Srinivasan et al., 2023).

Figure 11 shows the vertically integrated production terms at hour 31 of the weak-cooling simulation. All production terms are enhanced at sharp fronts, with shear production dominant. Areas of elevated finescale KE closely track regions of strong shear production, but are slightly broader presumably due to advection and lateral transport. The temporal evolutions of these energy sources averaged over sharp frontal regions are shown in Figure 12 for both simulations. Production terms in each simulation are normalized by the corresponding spatial mean of buoyancy production (the only source of turbulence) outside the front zone at hour 60. For convenience, we define the sharp frontal regions in each simulation as areas where the vertically integrated finescale KE exceeds 5×10^{-4} , and $1.2 \times 10^{-3} \text{ m}^3 \text{ s}^{-2}$, respectively. Other criteria based on horizontal buoyancy gradient, strain, and divergence can also be used, but do not change the results significantly. On average, frontal shear production is about an order of magnitude larger than buoyancy production in the weak-cooling simulation, and remains the largest source term even in the moderate-cooling case. Notably, at sharp fronts, the contributions from vertical shear production, strain-driven horizontal shear production, and convergence-driven horizontal shear production are approximately equal. The equipartition between $\Pi_{h,\sigma}$ and $\Pi_{h,-\delta}$ can be understood using the asymptotic theory presented in Srinivasan et al. (2023). Our analysis also shows the importance of vertical shear

production, especially under stronger forcing conditions (Figure 12b). This term, which is not adequately resolved in the RANS simulation analyzed by Srinivasan et al. (2023), represents another mechanism of submesoscale-induced turbulence through vertical momentum mixing (Zheng et al., 2025).

Overall, this energy budget analysis suggests that the localized finescale tracer subduction observed in sharp frontal regions is linked to submesoscale frontogenesis. The frontogenesis is promoted by both the convergence and strain of the submesoscale flow as the MLI-induced overturning circulation slumps the isopycnals. Along the sharpened front, the convergent and straining horizontal flow also facilitates downscale energy transfer to excite finescale motions, which then enhance the finescale tracer flux through stronger gradient production. Existing boundary layer parameterizations do not include the energy source from horizontal shear production, and would therefore underestimate the mixing and tracer transport at sharp fronts, even though the frontogenesis at scales larger than the grid scale could be partially resolved by submesoscale-permitting simulations. Under realistic conditions, the frontogenetic tendency may be quickly balanced by the frontolytic effect of finescale buoyancy mixing, but the downscale energy transfer could persist as long as the convergence and strain of submesoscale currents remain significant. Therefore, it is plausible that the enhanced finescale tracer subduction near sharp fronts may continue even after the frontogenesis is arrested by turbulence.

5. Implications for Finescale Parameterization at Fronts

Despite the enhanced finescale tracer subduction, finescale tracer flux is generally suppressed within the ML at fronts. To further examine these two major differences, and provide insights into parameterization of finescale tracer flux at submesoscale fronts, we diagnose an effective finescale diffusivity κ from the frontal-averaged vertical flux and vertical gradient,

$$\kappa = \frac{-\langle F_v' \rangle}{\partial \langle \bar{c} \rangle / \partial z}. \quad (28)$$

The angle bracket denotes horizontal averaging over the entire frontal zone, thus the diagnosed diffusivity should be interpreted in the context of coarse-resolution models that do not resolve any submesoscale processes.

The time-averaged nondimensional diffusivity profiles for the two forced simulations are shown in Figures 13a and 13b. Compared to the diffusivity in regions without fronts, both simulations exhibit weaker diffusivity within the ML but higher diffusivity below the ML base. This result has two implications. First, the suppression of finescale flux within the ML can not be fully explained by the change of the ML depth. This means that even though a coarse-resolution model may get the MLI-induced restratification right with current MLI parameterizations (Bodner et al., 2023; Fox-Kemper & Ferrari, 2008), its boundary layer mixing parameterization would still overpredict the diffusivity (gray lines in Figures 13a and 13b). Second, the enhanced finescale subduction at fronts is due to higher diffusivity, not simply because the vertical tracer gradient is different there.

The higher diffusivity, due to more efficient gradient production for finescale tracer flux, is ultimately linked to higher finescale KE near sharp fronts. As we have shown in Section 4.5, the primary energy source there is from the horizontal shear production associated with the frontogenetic process. Note that this differs substantially from the assumed frontal arrest by surface-forced turbulence scaling inherent in the parameterization of Bodner et al. (2023), in that frontal shear production overwhelms surface-forced turbulence as also found in Sullivan and McWilliams (2024). While Dong et al. (2024) correctly argue that shear production of various types becomes increasingly stronger as the frontal arrest scale is approached, the frontal arrest scale here is obtained by a different mechanism, perhaps because the restratification ratio R_{MLI} is relatively small in even the moderate-cooling case, and wind-driven turbulence is absent. The same is true of the regime simulated by Sullivan and McWilliams (2024). Existing parameterizations for boundary layer mixing cannot represent this frontogenesis-induced finescale tracer flux, as they assume the energy driving mixing is solely from surface forcing. For km-scale or even $\mathcal{O}(100 \text{ m})$ scale submesoscale-permitting RANS models, the frontogenetic process may be partially captured, but the associated downscale KE flux is likely dissipated and underestimated due to frontal arrest at the coarse grid scale. The results here indicate that instead frontally-energized turbulence should drive tracer transport; this is likely only possible if well-resolved fronts pass tracked levels of energy to the SGS and its tracer flux scheme.

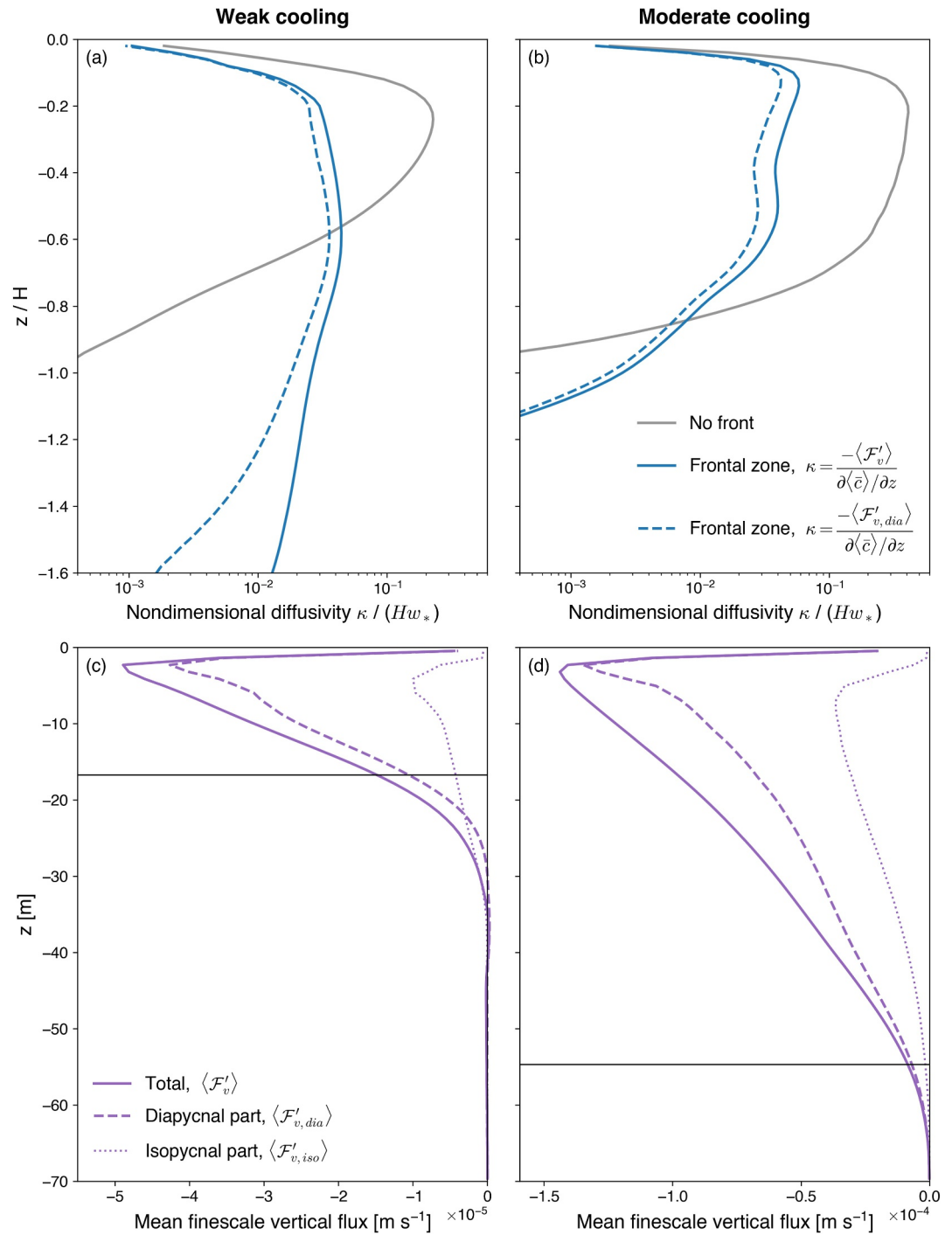


Figure 13. Profiles of (a and b) nondimensional finescale diffusivity, and (c and d) diapycnal–isopycnal decomposition of mean finescale vertical tracer flux (hours 14–47) in the frontal zone for simulation forced with weak and moderate surface cooling. Gray lines in panel (a and b) show the diagnosed diffusivity outside the frontal zone for reference. Horizontal lines in panel (c and d) indicate the mean mixed-layer depth.

A parameterization for this deep-reaching finescale tracer flux at submesoscale fronts could be developed if a corresponding parameterization for frontogenesis-induced KE dissipation is available. Recent work by Srinivasan et al. (2023) shows that the downscale KE flux across a certain length scale is proportional to the product of the convergence of larger scales and KE of smaller scales. This provides a potential scaling for the KE dissipation rate at equilibrium state ($\epsilon \sim \Pi \sim -\delta \mathcal{E}'$), which could then be combined with an assumed mixing efficiency (Chor

et al., 2022; Gregg et al., 2018; Osborn, 1980) to get an estimate for the diapycnal tracer diffusivity. However, this pathway is not immediately accessible due to several missing links in present practice. The main challenge is that this dissipation scaling requires knowledge of KE at subgrid scales, which encompasses both surface-forced turbulence and unresolved small-scale flows cascaded from larger scales. The detailed KE spectral transition from submesoscales to 3D isotropic turbulence under surface forcing is unknown.

Moreover, the aforementioned approach only represents diapycnal fluxes across density surfaces; any along-isopycnal fluxes would necessitate a different parameterization strategy (e.g., an anisotropic diffusivity as in Bachman et al., 2017). The partition of finescale vertical flux in diapycnal and isopycnal direction may also hold implications for future parameterization development. For instance, depending on if the isopycnal component is excluded, the diagnosed diffusivity can be quite different (see dashed lines in Figures 13a and 13b). Here we conduct such a decomposition by first projecting the 3D flux vector $\mathbf{F}'$ onto the direction of local buoyancy gradient, and then projecting onto the vertical direction, that is,

$$\mathbf{F}'_{v,dia} = \frac{\mathbf{F}' \cdot \nabla \bar{b}}{|\nabla \bar{b}|} \cdot \hat{\mathbf{k}}, \quad \mathbf{F}'_{v,iso} = \mathbf{F}' - \mathbf{F}'_{v,dia}, \quad (29)$$

where $\mathbf{F}'_{v,dia}$ and $\mathbf{F}'_{v,iso}$ are the diapycnal and isopycnal component of finescale vertical flux, respectively. The gradient is computed using the filtered buoyancy, as this is the information available for parameterizations. The filtered buoyancy surface generally moves slower than the finescale velocity, thus the effect of isopycnal heaving is considered small. A similar decomposition for the submesoscale vertical flux may not be appropriate as the influence of isopycnal movement becomes more important; in that case, the method proposed by Freilich and Mahadevan (2019) may be helpful. The time- and frontal-averaged flux components are shown in Figures 13c and 13d. On average, the finescale vertical tracer flux at submesoscale fronts is dominated by the diapycnal component in both simulations. In terms of the flux across the ML base, the dominance of the diapycnal component becomes more evident as the surface cooling increases. This somewhat surprising result, in light of the energetic ease of dispersing tracer along isopycnals rather than across, may be a signature of the shearing orientation imprinting on the turbulent structures, but it may also be sensitive to the limited spectral span of isotropic turbulence in these simulations (Figure 2)—specifically, it is possible that the Ozmidov scale is not well resolved, especially near the ML base. A smaller fraction of the vertical flux is along-isopycnal. This may indicate some contamination from MLI, but it may also reflect intrinsic characteristics of frontal turbulence, such as symmetric instability. More work is needed to clarify the mechanism of diapycnal and along-isopycnal flux, and their relative importance over a wider parameter space of forcing and frontal strength.

6. Summary and Discussion

Recent studies (e.g., Balwada et al., 2021; Freilich & Mahadevan, 2021; Mahadevan et al., 2020) have frequently emphasized strong surface–interior exchange at submesoscale fronts. A key challenge for understanding and quantifying this vertical exchange stems from the complex multiscale interactions between submesoscale processes and finescale turbulence at mixed-layer (ML) fronts. This study focuses on the case of submesoscale frontal spindown under varying surface cooling rates, with the goal of quantifying the role of finescale turbulence in subducting a surface-fluxed passive tracer in the presence of mixed-layer instability (MLI). The coarse-graining approach is employed to isolate the tracer transport by finescale turbulence and submesoscale processes in LES. Major conclusions from this investigation are summarized below:

1. Although the frontal-mean finescale tracer flux is generally suppressed in the ML relative to regions without fronts, it can penetrate below the ML depth and enhance tracer subduction (Figures 5c and 5f).
2. A carefully constructed tracer budget, using a control volume bounded by the evolving ML base, indicates that although submesoscale flux dominates the subduction, finescale tracer flux over just about 7% of the frontal zone area also contributes approximately 20% to the interior tracer accumulation (Figure 10).
3. Finescale tracer subduction is concentrated in frontogenetic regions with elevated finescale KE (Figures 9f–9h), which is primarily driven by the horizontal shear production associated with frontogenesis (Figures 11 and 12). Existing parameterizations would not be able to capture this mechanism (Figure 13).

Additionally, the spatial characteristics of vertical transport are analyzed using conditional averaging, revealing that surface submesoscale vorticity and strain spatially organize both the submesoscale and finescale tracer

transport into coherent subduction sites, where strain typically exceeds vorticity (Figure 6). A quadrant analysis also shows that finescale fluctuations in the frontal zone ML are less efficient in transporting tracer vertically compared to pure convective turbulence outside the fronts (Figure 8), presumably due to the mixture of shear instabilities and convective plumes.

By analyzing the energetics, this work offers an explanation of the localized finescale turbulence and the associated subduction. In essence, turbulence at fronts can be viewed as a combination of surface-forced and submesoscale-forced components, with the latter primarily associated with frontogenesis, although other mechanisms may also contribute (Taylor & Ferrari, 2010; Zheng et al., 2025). The surface-forced turbulence is largely confined to the ML, whereas the submesoscale-forced turbulence can penetrate below the ML base, thereby facilitating finescale subduction. The relative abundance of these two components likely modulates the role of finescale turbulence in the total subduction. With a larger share of the energy input from submesoscale processes, the weak-cooling case in our study indeed has a larger fraction of finescale contribution in the total subduction. This implies that finescale turbulence may play a more important role in the export of carbon produced by spring blooms than under fall conditions. This hypothesis should be tested in future research with simulations spanning a wider range of realistic parameter space. The highlighted finescale tracer subduction is also in agreement with recent work by Pham et al. (2024), which showed that rapid downwelling by finescale turbulence plays an important role in the subduction of near-surface particles at submesoscale fronts. There the localized intense turbulence at fronts were interpreted to be driven by surface forcing (convection). However, our work suggests an alternate interpretation of this point, as many submesoscale processes can also generate strong turbulence at these specific subduction sites.

The intensified finescale turbulence near frontogenetic regions may also play a role in supplying nutrients to the surface (e.g., entrainment rather than subduction). In fact, Whitt et al. (2019) have reported significantly stronger and deeper upward finescale nutrient flux during a storm with submesoscale fronts than without in a biogeochemical LES. However, whether the total entrainment is as efficient as the subduction examined here also depends on the interplay between submesoscale and finescale transport. For surface-rich tracers, subduction is promoted by the collocation of submesoscale downwelling and frontogenetic regions. For interior-rich tracers, entrainment by submesoscale flow and frontogenesis-enhanced finescale turbulence could occur in different locations, as the submesoscale upwelling regions are usually frontolytic. Nutrients entrained up by strong turbulence in frontogenetic regions may subsequently be advected or diffused downward by the submesoscale flow. This spatial mismatch could ultimately reduce the overall efficiency of upward nutrients transport. Future work on this problem would offer a more complete view of the multiscale surface–interior exchange.

Finally, other forms of submesoscale-turbulence interaction may also be important in shaping the multiscale tracer transport at fronts. For instance, surface forcing, especially downfront wind, is known to enhance the submesoscale flow (Hamlington et al., 2014; Verma et al., 2022; Whitt & Taylor, 2017) through stronger conversion of available PE. Our simulations similarly show a moderate increase of submesoscale buoyancy flux with surface cooling (Figures 5b and 5e). Surface-forced turbulent mixing can also modify the evolution of the frontal intensity by either inducing or prohibiting frontogenesis (Dauhajre et al., 2025), thereby feeding back on finescale subduction. How this interaction competes with MLI-induced frontogenesis has yet to be determined. On the other hand, because of the fast transport by turbulence, surface forcing also affects the amount of tracer available for the slower submesoscale flow to advect and diffuse. Note that the subduction of tracer in the unforced simulation is much smaller than those in the forced simulations (Figure 5). How the combined modulation of submesoscale flow and tracer gradients by surface forcing affects submesoscale tracer transport remains a key question for future research.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Availability Statement

The scripts for simulation setup, analysis are available in a Zenodo repository at <https://doi.org/10.5281/zenodo.21894759> (Zheng, 2026). The simulation outputs are too large to easily archive.

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